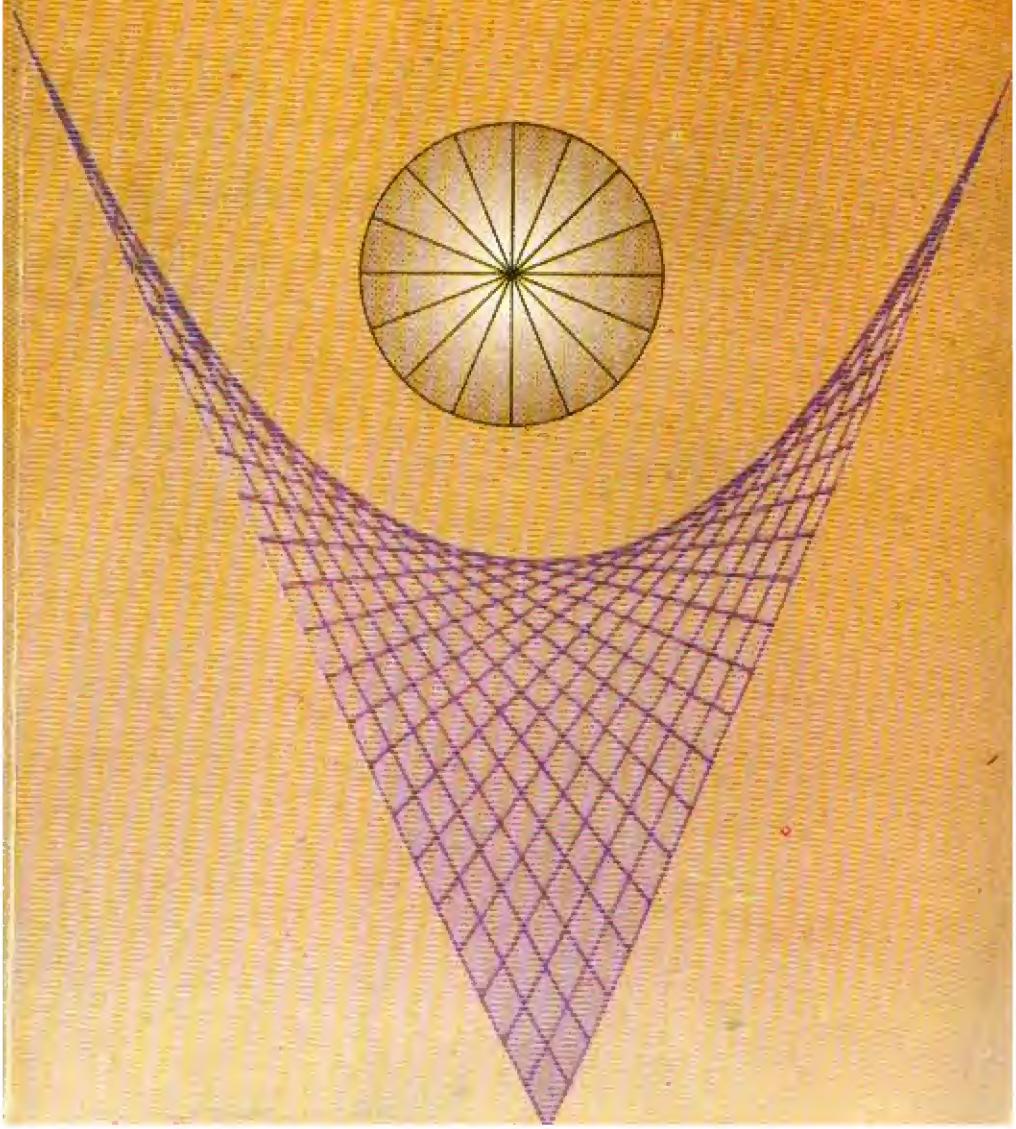
A Handbook for Designing Mathematics Laboratory in Schools



A Handbook

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Mathematics Laboratory

in Schools

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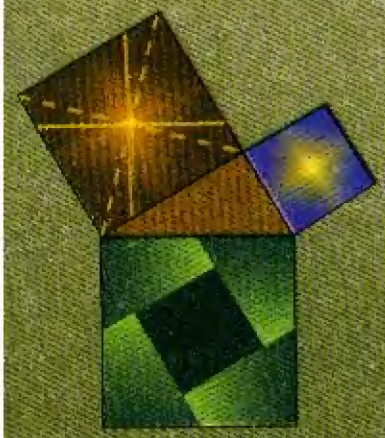

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

Preface

Mathematics has been used by almost all scientists, engineers and technologists. Its place in the sciences and in the practical arts, from the information and computational standpoints as well as its cultural significance, makes it indispensable in our life. We are striving to discover new and effective methods to educate our students. The concept of Mathematics Laboratory can be an important breakthrough in overcoming most of the instructional weaknesses in present day mathematics instruction.

It may be believed that a successful implementation of the Mathematics Laboratory concept will go a long way in removing maths phobia to our students.

The present handbook comprises six parts which deal in a precise manner the *know all* about Mathematics Laboratory. The issue of purpose and structure in one way and the requirement of equipments and activities at various stages in the other way are all dealt within the handbook. An attempt has also been made to answer questions which may arise in the minds of the teachers regarding practical marks in mathematics.

The present handbook is a modest beginning wherein an attempt has been made to make mathematics teaching as simple as possible for the benefit of the students, teachers and boards. It is hoped that once Mathematics Laboratory is established in the schools further improvement may be thought of by the individual schools.

It is a pleasure to express gratitude to all those who have been associated at all stages with the development of this handbook which was the need of the hour. Thanks are due to the Director, NCERT for his keen interest in the development of this handbook. The Joint Director, NCERT, Head, DESM and other colleagues are also duly acknowledged for giving their valuable suggestions which have resulted in the improvement of this handbook.

It is believed that the readers will find this handbook useful. Any suggestion from the readers for further improvement of the handbook will be highly appreciated.

Authors

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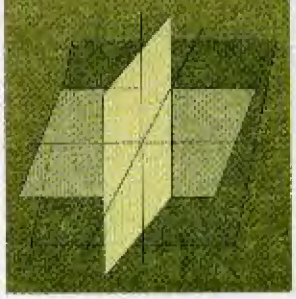
1 Introduction

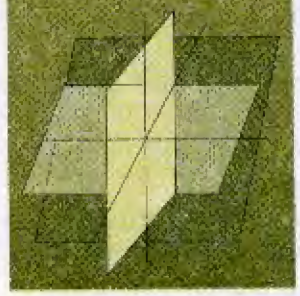


Mathematics has always occupied an important place in the school curriculum. The study of mathematics requires specific ability of intelligence. It helps in training and disciplining the mind as also developing the power of thinking and reasoning. Mathematical skills and their applications form an indispensable tool in our daily life. It has been rightly said that *higher the achievement in the field of mathematics, so is the nation's development.* Today, in the fast technologically-oriented society, it has found a greater role to play. There is hardly any subject in which mathematics has no role to play. However, inspite of all the above characteristics, due to the hierarchy of concepts and largely deductive and abstract nature of the subject, mathematics is considered as a very dull and difficult subject. A phobia has been created in the minds of the children that mathematics is tough to learn. As a result, most of the students are not taking interest in the subject and it has become one of the main causes of student's failure in mathematics. No doubt, mathematics is a subject which requires single-minded concentration and continuous efforts to achieve perfect and accurate learning. A student who can concentrate well can learn well.

Therefore, to remove this phobia, it is necessary to motivate the children by arousing and maintaining their interest in mathematics. This will create in them a positive attitude towards the subject. For this, it is necessary to make the involved abstractions tangible and concrete by developing the mathematical concepts out of direct personal experiences. It is a fact that most of the elementary and basic concepts on which the superstructure of modern mathematical theories rest, were evolved by the human beings out of direct personal experiences with concrete objects. If we analyse a mathematical problem, its comprehension and ultimate solution hinges around a correct perception of the elements of the physical situation involved in the problem. Therefore, by putting something concrete into the hands of a student, will give a better picture of conceptualisation of the problem. Hence, the traditional approach of teaching mathematics in which abstract concepts are usually presented to the students in an authoritarian way should be discouraged and activity approach which stresses the presentation of concrete experiences should be encouraged. Thus, mathematics has to be learnt by *doing* rather than by *reading*. This doing of mathematics gives rise to the need of a suitable place for performing these activities. A well-equipped *mathematics laboratory* is the suitable place for the same which can instantly motivate the students and create an environment to mathematics learning. This long felt need of mathematics laboratory was emphasised in *National Curriculum Framework for Elementary and Secondary Education (1988)* and has also been endorsed in the *National Curriculum Framework for School Education — 2000* (*NCFSE — 2000*) which states as follows:

> *To help exploration of mathematical facts through experimentation, a mathematics corner could be set up in the existing science laboratories. For this, existing science laboratories need to be converted into science-cum-mathematics laboratories.*

2 The Purpose of the Mathematics Laboratory



A mathematics laboratory can foster mathematical awareness, skill building, positive attitudes and learning by doing experiences in different branches of mathematics such as Algebra, Geometry, Mensuration, Trigonometry, Calculus, Coordinate Geometry, Mechanics, etc. It is the place where students can learn certain concepts using concrete objects and verify many mathematical facts and properties using models, measurements and other activities. It will also provide an opportunity to the students to do certain calculations using tables, calculators, etc., and also to listen or view certain audio-video cassettes, remedial instructions, enrichment materials, etc., of his/her own choice on a computer. Thus, it will act as an individualised learning centre for a student. It provides opportunities for discovering, remedial instruction, reinforcement and enrichment. Mathematics laboratory will also provide an opportunity for the teacher to explain and demonstrate many mathematical concepts, facts and properties using concrete materials, models, charts, etc. The teacher may also encourage students to prepare similar models and charts using materials like thermocol, cardboard, etc., in the laboratory. The laboratory will also act

as a forum for the teachers to discuss and deliberate on some important mathematical issues and problems of the day. It may also act as a place for teachers and the students to perform a number of mathematical celebrations and recreational activities. Thus, the purpose of a mathematics laboratory is to enable:

- a student to learn mathematics with the help of concrete objects and to exhibit the relatedness of mathematics with everyday life.
- a student to verify or discover some geometric properties using models, measurements, paper cutting, paper folding, etc.
- a student to use different tables and ready reckoners in solving some problems.
- a student to draw graphs and do certain calculations using computers and calculators.
- the students to do some field work like surveying, finding heights, making badminton courts, etc., using instruments kept in the laboratory.
- the students and teachers to organise mathematics club activities including celebration of birthdays of famous mathematicians.
- the students to listen or view certain audio or video cassettes, CDs relating to different mathematical concepts/ topics.
- a student to see a certain programme on a computer as a part of remedial instruction or enrichment under the proper guidance of the teacher.
- the students to perform certain experiments, which can be easily evaluated by the teacher.
- the students to do certain projects under the proper guidance of the teacher.
- the students to perform certain recreational activities in mathematics.
- a teacher to visually explain some abstract concepts by using three-dimensional models.
- a teacher to demonstrate certain concepts and patterns using charts and models.
- a teacher to demonstrate and reinforce the truth of certain algebraic identities using different models.
- a teacher to demonstrate the truth of various formulae for areas and volumes of different plane and solid figures using models.

- a teacher to explain certain concepts using computers and calculators.
- the teachers and students to consult good reference mathematics books, journals, etc., kept in the laboratory.
- the teachers to meet and discuss important issues relating to mathematics from time to time.
- a teacher to explain certain concepts, data, graphs, etc., using slides.
- a teacher to generate different sets of parallel tests using a computer for testing the achievement of students.
- the budding mathematicians to take inspiration from the lives, works and anecdotes relating to great mathematicians.

3 Equipment for the Mathematics Laboratory

As the students will be involved in a lot of model making activities under the guidance of the teacher, the smooth running of the mathematics laboratory will depend upon the supply of number of oddments. These include string and thread, sealing wax and soft red wax, cello tape, white cardboard, hardboard, needles and pins, drawing pins, sandpaper, pliers, screw-drivers, rubber bands, gummed papers and labels, matches, squared paper, plywood, scissors, saw, paint, solder iron, solder wire, iron wire, cotton wool, tin, plastic sheet, glazed papers, etc. Besides these, some models, charts, slides, etc., made up of a good durable material should also be there for the teacher to demonstrate some mathematical concepts, facts and properties before the students. Different tables, ready recknors, should also be there in the laminated form so that these can be used by the students for different purposes. Further, for performing activities such as measuring, drawing, and calculating, consulting reference books, etc., there should be equipments like mathematical instruments, calculators, computers, books, journals etc., in the laboratory. In view of the above, following is the list of equipments suggested for the laboratory.

3.1 INSTRUMENTS

Mathematical instrument set (Ruler, Set-squares, Divider, Protractor and Compasses), metre scale, measuring tape, diagonal scale, calculators, computers, etc.

3.2 TABLES

- Squares
- Square roots
- Cubes
- Cube roots
- Compound Interest
- Recurring Deposits

3.3 READY-RECKONERS

- Wages
- Interest
- Taxes

3.4 CHARTS, PICTURES

Charts of the following (involving numbers 1 to 100)

- Even and odd demarcated with different colours
- Multiples of three demarcated
- Multiples of five demarcated
- Primes demarcated
- Multiples of nine demarcated
- Charts of units of length, area, volume along with their abbreviations
- Charts of magic squares, important formulae, regional numerals, numerals of ancient times and portraits of mathematicians along with anecdotes.

3.5 MODELS

- Number line
- Geoboards – rectangular, circular and isometric
- Models for verifying the following identities:

(i) $(a + b)^2 = a^2 + 2ab + b^2$

(ii) $(a - b)^2 = a^2 - 2ab + b^2$

(iii) $(a + b)(a - b) = a^2 - b^2$

(iv) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(v) $k(a + b + c) = ka + kb + kc$

(vi) $(a+b)^2-(a-b)^2=4ab$
(vii) $(a+b)^2+(a-b)^2=2(a^2+b^2)$
(viii) $(a+d)(b+c)=ab+ac+db+dc$
(ix) $(a+b)(a+c)=a^2+a(b+c)+bc$
(x) $acx^2+(ab+cd)x+bd=(ax+d)(cx+b)$

- Models for verifying the following identities:
  (i) $(a+b)^3=a^3+3a^2b+3ab^2+b^3$
  (ii) $(a-b)^3=a^3-3a^2b+3ab^2-b^3$

- Models of
  (i) Equilateral triangle
  (ii) Isosceles triangle
  (iii) Scalene triangle
  (iv) Right triangle
  (v) Square
  (vi) Rectangle
  (vii) Quadrilateral
  (viii) Parallelogram
  (ix) Rhombus
  (x) Trapezium
  (xi) Regular pentagon
  (xii) Regular hexagon
  (xiii) Regular octagon
  (xiv) Circle
  (xv) Right angle
  (xvi) Acute angle
  (xvii) Obtuse angle
  (xviii) Linear pair
  (xix) Supplementary angles
  (xx) Complementary angles.

- Models of
  (i) Cube
  (ii) Cuboid
  (iii) Tetrahedron
  (iv) Right circular cylinder
  (v) Right circular cone
  (vi) Right prisms, Pyramids with triangular, square and hexagonal bases
  (vii) Sphere and hemisphere.

- Hollow hinged models of the items listed above (made of tin sheets)
- Model for conic sections
- Models for finding the centre of a circle
- Models for verifying the following results:
  - (i) Sum of the angles of a triangle is 180°.
  - (ii) Sum of the angles of a quadrilateral is 360°.
  - (iii) Sum of exterior angles of a polygon is 360°.
  - (iv) Opposite angles of a parallelogram are equal.
  - (v) Opposite angles of a cyclic quadrilateral are supplementary.
  - (vi) Corresponding angles/alternate angles made by two parallel lines with a transversal are equal.
  - (vii) Exterior angle of a triangle is equal to the sum of the two interior opposite angles.
  - (viii) Angles in the same segment of a circle are equal.
  - (ix) Angle at the centre is double the angle at any point on the circle.
  - (x) Equal chords are equidistant from the centre and its converse.
  - (xi) Tangents from an external point to a circle are equal.
  - (xii) If two circles touch each other, then the two centres and the point of contact are collinear.
  - (xiii) Base angles of an isosceles triangle are equal and its converse.
  - (xiv) Inequality results in a triangle.
  - (xv) Vertically opposite angles are equal.
  - (xvi) Diagonals of a parallelogram bisect each other.
  - (xvii) Diagonals of a rhombus bisect each other at right angles.
- Models illustrating following concepts/properties:
  - (i) Concurrency of medians of a triangle
  - (ii) Concurrency of altitudes of a triangle
  - (iii) Concurrency of angle bisectors of a triangle
  - (iv) Concurrency of perpendicular bisectors of the sides of a triangle
  - (v) Incircle, circumcircle and excircles in a triangle
  - (vi) Height and slant height of a cone
  - (vii) Height and slant height of a pyramid
  - (viii) Angles forming linear pair
  - (ix) SSS congruency of triangles

(x) ASA congruency of triangles
(xi) SAS congruency of triangle
(xii) RHS congruency of triangles
(xiii) Angles in a semicircle
(xiv) Major segment and minor segment of a circle.

- Models for verifying Pythagoras theorem by:
  (i) Counting small squares
  (ii) Baudhayana's method
  (iii) Bhaskara's method
  (iv) Cutting the square on hypotenuse into four congruent quadrilaterals and a square.
- Models for showing relationship between area of:
  (i) Parallelogram and a rectangle on the same base and between the same parallels.
  (ii) A triangle and that of a rectangle.
  (iii) A parallelogram and that of a trapezium.
  (iv) Two triangles on the same base and between the same parallels.
- Models of hollow prisms and pyramids of the same base and same height of three different sizes.
- Models of hollow cylinders and cones of the same base and same height of three different sizes.
- Models for deriving formulae for area/volume:
  (i) Circles sliced into sectors for area of a circle.
  (ii) Sphere sliced along diameters which can be rearranged to form a cylinder for surface area.
  (iii) Sphere sliced into pyramids with vertices at the centre for the volume of the sphere.
- Hinged models for demonstrating symmetry of :
  (i) Square
  (ii) Rectangle
  (iii) Isosceles triangle
  (iv) Equilateral triangle
  (v) Circle
- Model for demonstration of :
  (i) Sets
  (ii) Relations and functions

(iii) Quadratic functions with the help of linear functions
(iv) Sequence and series
(v) Height and distance
(vi) Pascal's triangle
(vii) Arithmetic progression
(viii) Conic sections
(ix) Increasing, decreasing functions
(x) Maxima, minima, point of inflection
(xi) Lagrange's mean value theorem
(xii) Rolle's theorem
(xiii) Definite integral as limit of sum
(xiv) Angle in a semicircle using vectors
(xv) Moment of a force about a point
(xvi) Moment of couple
(xvii) Construction of parabola when distance between directrix and focus is given
(xviii) Construction of ellipse when major and minor axes are given
(xix) Octants
(xx) Shortest distance between two lines
(xxi) Geometrical interpretation of scalar triple product
(xxii) Equation of a straight line passing through a fixed point and parallel to a given vector
(xxiii) Equation to a plane
(xxiv) Angle between two planes
(xxv) Bisection of the angle between two planes by a third plane
(xxvi) Intersection of three planes
(xxvii) Sphere and its plane sections
(xxviii) Normal probability curve and area under the normal probability curve
(xxix) Projection of the line segment
(xxx) Application of the Lami's theorem
(xxxi) Centre of parallel forces

3.6 Overhead Projector along with Slides

3.7 Audio and Video Cassettes, C.D.

3.8 Calculators

3.9 Computers

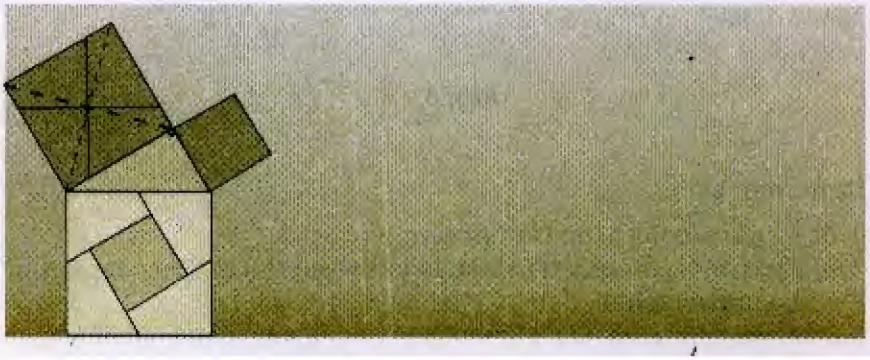
3.10 Books and Journals

3.11 Portraits of Mathematicians

4 Activities at Different Stages

As already stated, students will be performing a number of model/chart making activities under the guidance of the teacher in the laboratory. In order to help students and teachers in this task, some such activities are suggested at different stages of school education in this section along with their preparation and demonstration. The teacher may help the students to prepare these and some more activities on their own using material mentioned in the preceding section.

4.1 Activities at the Elementary Stage



ACTIVITY 1

Base Value and Place Value

Materials

Wooden cubes, glaze papers, cardboard base, tape, sketch pen, scissors.

Preparation

(i) Take a cardboard base

(ii) Cover it with glaze paper.

(iii) Take a cube and paste it on the base. It is a unit (Fig. 4.1.1 a).

(iv) Make a cuboidal rod of 10 cubes by fixing cubes on the above paper. Paste the rod on the base (Fig. 4.1.1 b)

(v) Take 10 cuboidal rods of the type as in step (iv) above and paste them on the base (Fig. 4.1.1 c).

(vi) Make 10 solids of the type as in step (v) above and paste them on the base (Fig. 4.1.1 d).

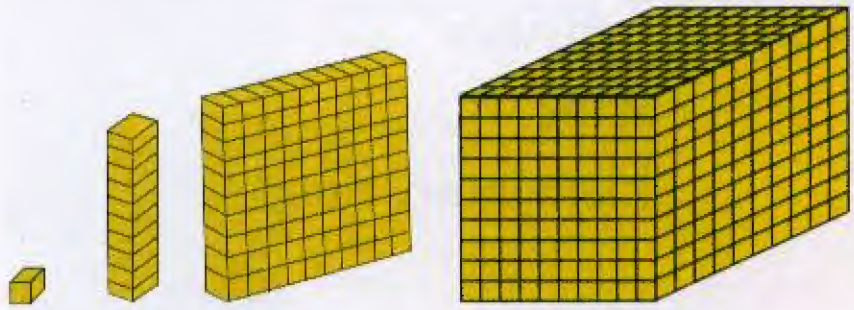


***Fig.** 4.1.1*

Demonstration

(i) 1 cube depicts the unit or one.

(ii) Ten units combined to form a cuboidal rod of tens, depicting 10 units =1 ten.

(iii) Ten tens combined to form a solid of 100 units or ten tens, depicting
100 units = 1 hundred
i.e. 10 tens = 1 hundred

(iv) Ten hundreds combined together to form a cube of $10 \times 10 \times 10$. It shows:
1000 units = 1 thousand
100 tens = 1 thousand
10 hundred = 1 thousand

Use

- Model can be used to explain the idea of base value and place value.

ACTIVITY 2

Number Plates

Materials

Transparent sheets, scale, scissors, marker pen.

Preparation

(i) Cut 2.5 cm × 2.5 cm square as single unit from the transparent sheet. Cut out 12 such units.

(ii) Cut out 2.5 cm × 5 cm rectangle as double unit. Cut out 6 such units.

(iii) Cut out 2.5 cm × 7.5 cm rectangle as triple unit and cut out 4 such units.

(iv) Similarly, cut out units (strips) of sizes 2.5 cm × 10 cm, 2.5 cm × 12.5 cm, 2.5 cm ×15 cm, 2.5 cm × 17.5 cm, 2.5 cm × 20 cm, 2.5 cm × 22.5 cm, 2.5 cm × 25 cm, 2.5 cm × 27.5 cm, 2.5 cm × 30 cm.

(v) Write the numbers on these plates obtained as follows:

2.5 cm × 2.5 cm plates as 1

2.5 cm × 5 cm plates as 2

2.5 cm × 7.5 cm plates as 3, etc. (Fig. 1.1.2 a)

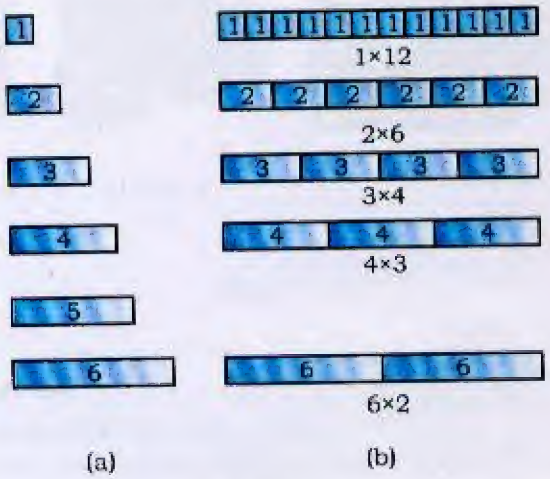


Fig. 4.1.2

Demonstration

Working is as shown below:

(i) Show these plates in the classroom one by one to give the idea of natural numbers.

(ii) Place two or more number plates adjacent to one another and give the idea of successor of a number and the addition of numbers by comparing the result with some other number plate.

(iii) Place one number plate over another to give the idea of predecessor of a number and also of the subtraction of numbers.

(iv) Idea of factors and multiples can be explained as follows:

Take the number plates of 1, 2, 3, 4, 5, 6, etc. It can be observed that number plate of 12 can be covered with 12 plates of number 1, 6 plates of number 2, 4 plates of number 3, 3 plates of number 4, 2 plates of number 6, thereby showing that 1, 2, 3, 4, 6, 12 are factors of 12 (Fig. 4.1.2 b).

(v) Similarly, from the above we can also show that 12 is a multiple of 1, 2, 3, 4, 6 and 12.

(vi) Idea of H.C.F. can also be given with these plates by taking two or three bigger plates.

Use

- Model can be used to introduce natural numbers
- It can also be used to explain the idea of factors, multiples, HCF and LCM.

ACTIVITY 3

Magic Square of the Type 3×3 of Magic Constant 15

Materials

Transparent sheet, adhesive, coloured paper, sketch pen, scissor.

Preparation

(i) Take a square transparent sheet of size 12cm × 12cm.

(ii) Make a 3 × 3 square on the transparent sheet.

(iii) Put the number $\frac{15}{3}$, i.e. 5, called central number, in the middle cell.

(iv) Add 2 to the central number and subtract 2 from the central number and put these numbers to the right and left of the central number, respectively along the central row (Fig. 4.1.3a).

(v) Add 1 to the central number and subtract 1 from the central number and put these numbers upward and downward, respectively along the right diagonal (Fig. 4.1.3b).

(vi) Add 3 to central number and subtract 3 from the central number and put these numbers upward and downward, respectively along the left diagonal (Fig. 4.1.3c).

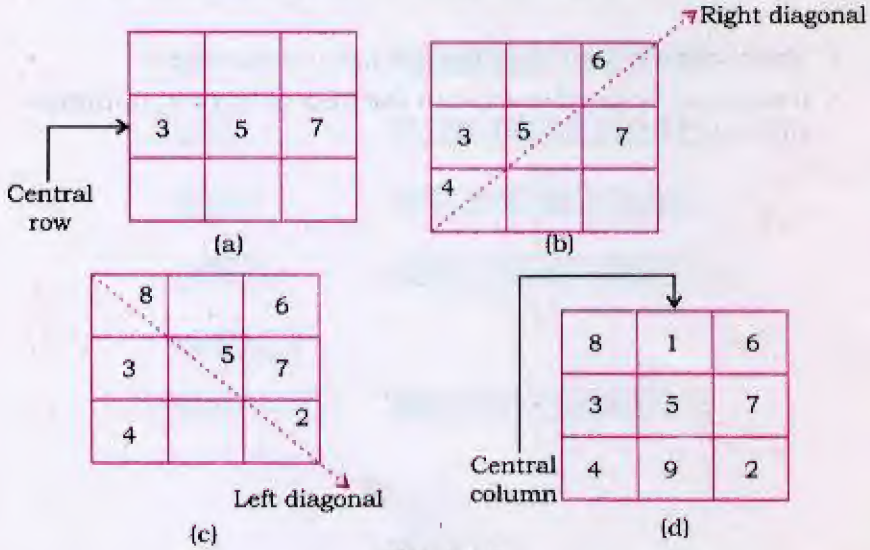


Fig. 4.1.3

(vii) Subtract 4 from the central number and add 4 to the central number and write these numbers upward and downward, respectively along the central column (Fig. 4.1.3d).

Performing these activities step wise, magic square of magic constant 15 is obtained. Steps involved have been shown in the figure 4.1.3.

Demonstration

(i) The sum of numbers in each row is 15.

(ii) The sum of number in each column is 15.

(iii) The sum of number in each diagonal is 15.

Use

- Model can be used to explain magic square of magic constant as multiples of 3.

ACTIVITY 4

Magic Square of the Type 4 × 4 of Magic Constant 34

Materials

Transparent sheet, adhesive, coloured paper, sketch pen, scissor.

Preparation

(i) Take a square transparent sheet of size 12cm × 12cm.

(ii) Cut a 4 × 4 square from the transparent sheet.

(iii) Write the numbers 1 to 16 in an order in the squares and mark the squares with identical shapes in which the numbers are to be interchanged (Fig. 4.1.4a).

(iv) Interchange the numbers in the squares as discussed above and obtain the required 4 × 4 magic square as shown in figure 4.1.4b.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

(a)

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

(b)

Fig. 4.1.4

Demonstration

(i) Demonstrate that the sum of numbers taken along any row, column or diagonal remains constant, i.e., number 34.

(ii) Demonstrate the above stated rule to obtain magic square of the type 4 × 4 using any consecutive 16 natural numbers viz. from 15 to 30, or from 30 to 45, 45 to 60, etc. and find their corresponding magic constant.

Use

- Model can be used to explain magic square of magic constant 34 and other magic constants as demonstrated above.

ACTIVITY 5

HCF by Division Method

Materials

Thermocol sheet, 1 unit wide cardboard strips, coloured paper, glue, scissors, scale, pencil, etc.

Preparation

(i) Take a thermocol sheet of about 50 units × 50 units and cover it with white paper.

(ii) Cut 1 unit wide cardboard strips to get 2 pieces of 35 units, 3 pieces of 20 units, 3 pieces of 15 units, 4 pieces of 5 units length.

(iii) Stick the cardboard strips as shown in the figure 4.1.5.

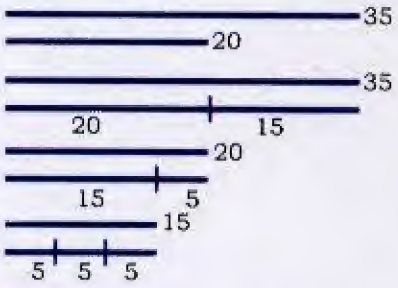


Fig. 4.1.5

Demonstration

(i) The first set of strips represent the numbers 35 and 20 whose HCF is to be found.

(ii) The second set of strips shows the division of the larger number by the smaller number i.e. $35 = 20 \times 1 + 15$.

$$\begin{array}{r|l} 20 & 35 \\ & 20 \\ \hline & 15 \end{array} \quad 1$$

(iii) The third set of strips shows the division of 20 by 15. Remainder 5 is the next divisor.

$$\begin{array}{r|l} 15 & 20 \\ & 15 \\ \hline & 5 \end{array} \quad 1$$

(iv) Fourth set of strips shows the division of 15 by 5, i.e., $15 = 5 + 5 + 5$

$$\begin{array}{r|l} 5 & 15 \\ & 15 \\ \hline & 0 \end{array} \quad 3$$

The required HCF of 35 and 20 is 5 (the last divisor in the process when the remainder is zero).

5 units strip can measure 35 units and 20 units strips an exact number of times, i.e., the numbers 20 and 35 are exactly divisible by 5.

Use

- Model can be used to explain the methods of finding HCF of two numbers by continued division.

ACTIVITY 6

Lowest Common Multiple

Materials

Hardboard, chart paper, scale, pencil, eraser, sketch pens, scissors, saw etc.

Preparation

(i) Cut a 15 units × 12 units hardboard. This will be the base board.

(ii) Cut 12 units × 12 units hardboard and then cut 5 more boards of this measurement.

(iii) All the boards will have a margin of 1 unit width on all the four sides.

(iv) On the base board, we make a 10 units × 10 units square and make equal squares on it. (Fig. 4.1.6 a)

(v) We paste the heading on top of the base board and write numbers 1 to 100 on these squares.

(vi) On the other 5 boards, we make 100 equal squares leaving the margin. (Fig. 4.1.6 b)

(vii) On each board, represent the multiples of different numbers, such as 2, 3, 4, 5, 6, etc.

(viii) Drill the squares which represents the multiples of the number specified.

(ix) We paste to the respective headings written on the chart paper on these boards as follows:

Multiples of 2

Multiples of 3

Multiples of 4

Multiples of 5

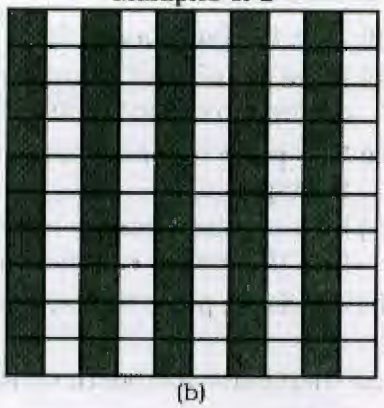
Multiples of 6

Base Board

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

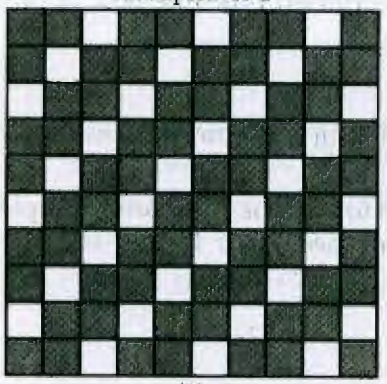
(a)

Multiples of 2



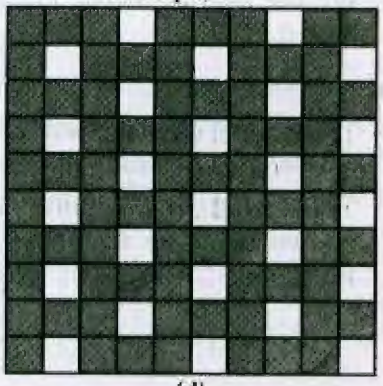
(b)

Multiples of 3



(c)

Multiples of 4



(d)

Multiples of 5

(e)

Multiples of 6

(f)

Fig. *4.1.6*

Demonstration

(i) Take the base board first.

(ii) Place the second board on it which shows the multiples of 2 on the top of the second board.

(iii) Now place the third board which shows the multiples of 3 on the top.

(iv) Now see the common multiples of 2 and 3 and we can easily find the lowest common multiple of 2 and 3.

(v) Similarly, place the next board, i.e., the multiples of 4 on top of it.

(vi) Then place the board containing multiples of 5, multiples of 6, etc.

(vii) We see that 60 is the lowest common multiple of 2, 3, 4, 5 and 6.

Use

- This model can be used in teaching the concept of multiples of specified numbers.
- This model can be mainly used to find the common multiples of two or three numbers and then their lowest common multiples.

ACTIVITY 7

Fractional Kit

Materials

Hardboard, cardboard, glaze paper, adhesive, scale, sketch pen, scissors.

Preparation

(i) Cut out a rectangle from the cardboard sheet.

(ii) Paste a full cardboard sheet under the remaining portion of the cardboard got in step (i).

(iii) Cut-out rectangle is a unit which can be fixed in the cardboard frame made in step (ii).

(iv) With the help of cardboard, make different parts of the rectangular unit like half (vertically), half (horizontally), one-third, two-thirds, one-fourth, two-fourths, three-fourths, one-sixth, two-sixths, three-sixths, like wise one-eight, two-eights, etc.

(v) Make a pocket at the back of the model to keep parts of the model.

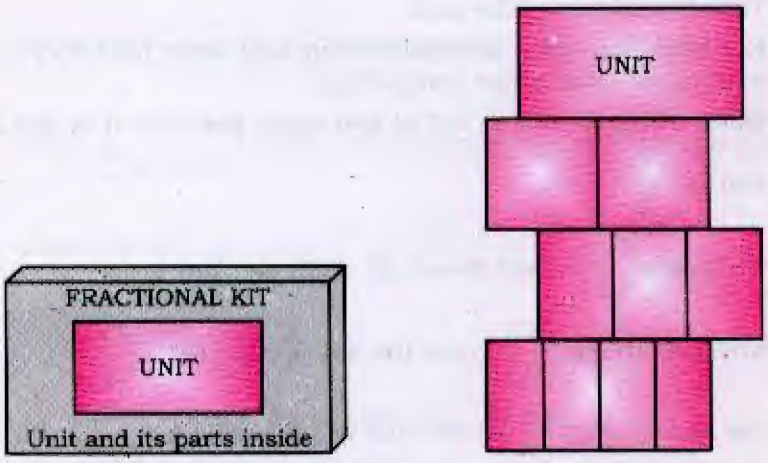


Fig. 4.1.7

Demonstration

This model demonstrates

(i) the verification of the result

$$\frac{2}{2}=\frac{3}{3}=\frac{4}{4}=\frac{5}{5}=\frac{6}{6} \ldots\ldots\ldots\ldots\ldots\ldots = 1$$

(ii) the concept of equivalent fractions like

$$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{6}{12}, \text{ etc.}$$

(iii) addition and subtraction of fractions having same denominators like

$$\frac{1}{2}+\frac{1}{2} \text{ and } \frac{1}{3}+\frac{2}{3}, \text{ etc.}$$

(iv) comparison of fractions like

$$\frac{1}{2}>\frac{1}{3}>\frac{1}{4}>\frac{1}{5}, \text{ etc}$$

(v) multiplication of fractions like

$$\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}, \text{ etc.}$$

and many other properties relating to fractions.
It can be done as follows:

A. (i) Take two halves of the unit.

(ii) Put both half parts simultaneously and show that they and unit cover each other completely.

(iii) Since one part is one out of two equal parts so it is one by two i.e. $\frac{1}{2}$.

B. Two halves $\left(\frac{1}{2}\right)$ cover the whole unit, i.e., $\frac{2}{2} = 1$.

Three one-thirds $\left(\frac{1}{3}\right)$ cover the whole unit, i.e., $\left(\frac{3}{3}\right) = 1$.

Four one-fourths $\left(\frac{1}{4}\right)$ cover the whole unit, i.e., $\left(\frac{4}{4}\right) = 1$.

Six one-sixths $\left(\frac{1}{6}\right)$ cover the whole unit, i.e., $\left(\frac{6}{6}\right) = 1$.

C. Take $\left(\frac{1}{2}\right)$ of the unit.

Put two $\left(\frac{1}{4}\right)$ of the unit on $\left(\frac{1}{2}\right)$ unit as above.

They will cover each other completely, which shows that

$\frac{1}{2}=\frac{1}{4}+\frac{1}{4}=\frac{2}{4}$

Likewise, we can show that

$\frac{1}{2}=\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{3}{6}$ and so on.

D. Take two one-thirds and put them together, i.e., $\frac{1}{3}+\frac{1}{3}$.

These are two parts out of three equal parts. So these are equal to $\frac{2}{3}$.

Hence, $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$

Likewise $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{3}{4}$, and so on.

E. Take $\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{6},\frac{1}{8}$ and on comparing by placing one over the other, we will find that $\frac{1}{2}>\frac{1}{3}>\frac{1}{4}>\frac{1}{6}>\frac{1}{8}$.

F. Take $\frac{1}{2}$ of the unit $\frac{1}{3}$.

$\frac{1}{2}\times\frac{1}{3}$ = one half of one-third i.e. one sixth

$=\frac{1}{6}$.

Likewise, we can show that $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$.

$\frac{1}{2} \times \frac{2}{4} = \frac{2}{8}$,

$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$, etc.

Use

- Model can be used to explain the concepts of fractions, their comparison and operation on them.

ACTIVITY 8

Algebraic Identity : $(a + b)^2 = a^2 + 2ab + b^2$

Materials

Thermocol sheet, adhesive, thermocol cutter, glaze papers, sketch pens.

Preparation

(i) Cut out two squares of sides a and b units such that $a > b$ from a thermocol sheet (a can be taken as 7 cm and b as 4 cm).

(ii) From the same thermocol sheet, cut out two rectangles of length a and breadth b units respectively.

(iii) The above cut outs can be fixed with the help of the adhesive as shown in figure 4.1.8.

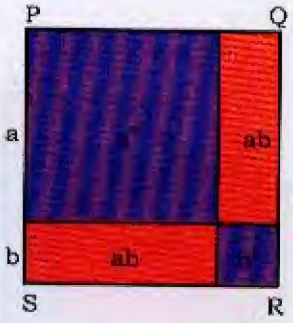


***Fig.** 4.1.8*

Demonstration

Model clearly demonstrates the algebraic identity:

$(a + b)^2 = a^2 + 2ab + b^2$

It is because area of square PQRS = $(a + b)^2$ and it is also equal to the sum of, a^2 , 2 times ab and b^2 .

Use

- Model can be used to verify the algebraic identity: $(a+b)^2 = a^2 + 2ab + b^2$, geometrically.

ACTIVITY 9

Algebraic Identity : $(a - b)^2 = a^2 - 2ab + b^2$

Materials

Cardboard, glaze paper, scissors, sketch pen, scale, adhesive.

Preparation

(i) Take a square with side a. Its area is a^2.

(ii) Take a smaller square with side b. Its area is b^2.

(iii) Take a square of side $(a - b)$. Its area is $(a - b)^2$.

(iv) Take two rectangles of length a and breadth b. Area of each is ab.

(v) Keep smaller square (b^2) on the right corner of bigger square (a^2) as shown in figure 4.1.9.

(vi) Make a pocket at the back of the model to keep the parts of the model.

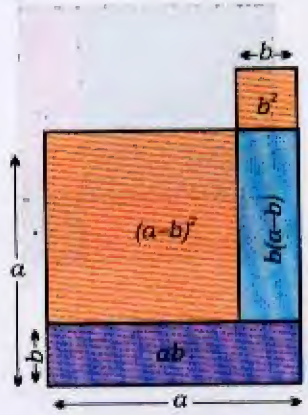


Fig. 4.1.9

Demonstration

(i) On the model, show a^2 and b^2.

(ii) Take the square of side $(a-b)$ from the pocket (at the back) of the model and put it on a^2. The area of this square will be $(a - b)^2$ as one side of the square is $(a - b)$.

(iii) This square $(a - b)^2$ will be equal to sum of squares a^2 and b^2 minus two rectangles of area ab from this sum.

(iv) Explain that

$a^2 + b^2 = (a - b)^2 + b^2 + b(a - b) + ab$

$= (a - b)^2 + 2ab$

(v) Thus, we get

$(a - b)^2 = a^2 - 2ab + b^2$

Use

- Model can be used to verify the algebraic identity $(a - b)^2 = a^2 - 2ab + b^2$, geometrically.

ACTIVITY 10

Algebraic Identity : $a^2 - b^2 = (a + b)(a - b)$

Materials

Cardboard, glaze paper, scissors, sketch pen, scale, adhesive.

Preparation

(i) Take a cardboard base.

(ii) Paste a square with side a on the base. Area of this square is a^2.

(iii) Make one more square with side b such that $b < a$. The area of this square is b^2.

(iv) Paste smaller square b^2 on the bigger square a^2.

(v) Cut the remaining portion of the square a^2 ($a^2 - b^2$) in the form of two trapezia as shown in the figure 4.1.10b.

(vi) Make a pocket at the back of the model to keep the parts of the Model.

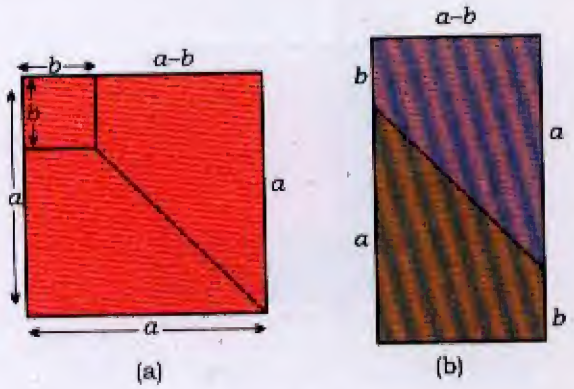


Fig. 4.1.10

Demonstration

(i) Put the smaller square with side b and area b^2 on the bigger square.

(ii) Join the remaining two trepezia covering area of $a^2 - b^2$ in such a way that a rectangle is formed as shown in figure 4.1.10b.

(iii) The breadth and length of this rectangle are $(a-b)$ and $(a+b)$. Thus, its area is $(a-b)(a+b)$.

(iv) In this way, it can be verified that

$a^2 - b^2 = (a-b)(a+b)$

Use

- Model can be used to verify the algebraic identity:

  $a^2 - b^2 = (a-b)(a+b)$, geometrically.

ACTIVITY 11

Algebraic Identity : $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Materials

Hardboard, adhesive, water colours, knobs of wood etc.

Preparation

(i) Take a hardboard to use as base.

(ii) Make a boundary of a square of side 18 units using hardboard strip.

(iii) Divide this square into squares/rectangles using the dimensions a = 10 units, b = 6 units and c = 2 units as shown in the following figure.

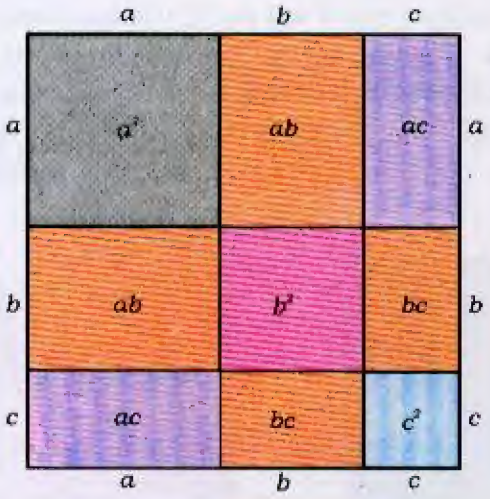


Fig. *4.1.11*

Demonstration

This model verifies the identity :
$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$,
which can be easily seen in figure 4.1.11.

Use

- Model can be used to verify the algebraic identity:
  $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$, geometrically.

ACTIVITY 12

Parallel Lines Intersected by a Transversal

Materials

Thermocol base, glaze papers (2 colours), drawing pins, steel wires, sketch pens and adhesive.

Preparations

(i) Take a thermocol base of size 16 units × 20 units.

(ii) Cover it with a coloured sheet of glaze paper. Cut out a margin of a different colour and paste it on all the four sides.

(iii) Cut out three steel wires and arrange two of them in such a way that they are parallel to each other and keep the third in such a way that it intersects the other two.

(iv) Cut out angles 1, 2, 3, 4 of different colours and paste them as shown in the figure 4.1.12.

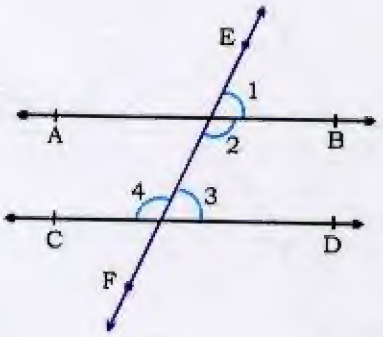


Fig. 4.1.12

Demonstration

Working is as follows:

(i) Measure $\angle 1$ and $\angle 3$. It can be verified that $\angle 1 = \angle 3$; i.e., a pair of corresponding angles are equal.

(ii) Measure $\angle 2$ and $\angle 4$. It can be verified that $\angle 1 = \angle 4$; i.e., a pair of alternate interior angles are equal.

(iii) Measure $\angle 2$ and $\angle 3$. It can be verified by adding the 2 that $\angle 2 + \angle 3 = 180°$. Thus, a pair of interior angles on the same side of the transversal are supplementary. Thus, it can be observed that if two parallel lines are intersected by a transversal, then each pair of corresponding angles and alternate interior angles are equal and the interior angles on the same side of the transversal are supplementary.

Use

This model can be used to demonstrate that if two parallel lines are intersected by a transversal, then

- each pair of corresponding angles are equal.
- each pair of alternate interior angles are equal.
- each pair of interior angles on the same side of the transversal are supplementary.
- vertically opposite angles are equal.
- linear pair can be shown.

ACTIVITY 13

Sum of the Angles of a Triangle

Materials

Thermocol sheet, hardboard sheet, glaze papers, sketch pens, adhesive.

Preparation

(i) Take a thermocol sheet and draw a (scalene) triangle on it and name it as ΔABC.

(ii) Colour the three angles of this triangle with the help of glaze papers of different colours.

(iii) Now take a hardboard sheet and cut out the angles with measures the same as of ∠A, ∠B, ∠C and of the same colour as used in the triangle.

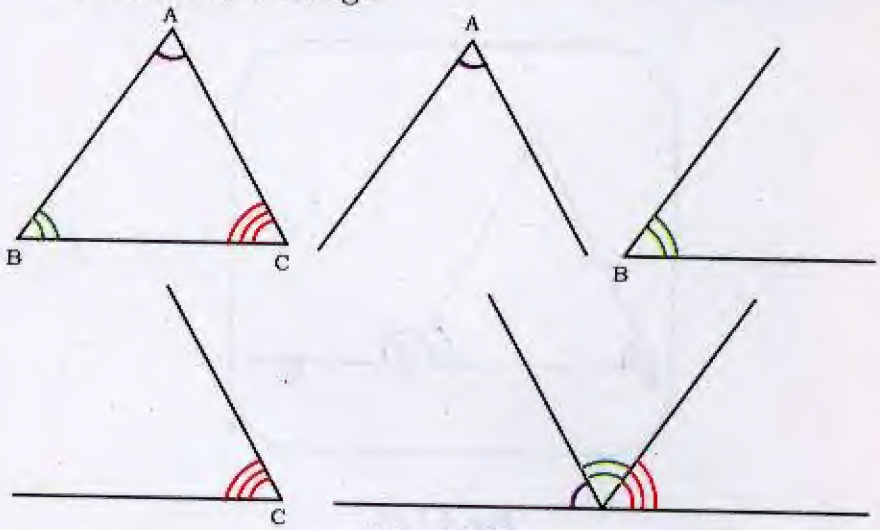


Fig. 4.1.13

Demonstration

The cut outs of the three angles A, B and C can be placed adjacent to each other at a point. They will form a line. It shows that sum of the three angles of a triangle is 180°.

Use

- Model can be used to verify that sum of the three angles of a triangle is 180°.

ACTIVITY 14

Exterior Angle of a Triangle

Materials

Wire, a piece of thermocol sheet, coloured glaze paper, drawing pins, solder iron, adhesive, protractor.

Preparation

(i) Take a piece of thermocol sheet and cover it with coloured glaze paper.

(ii) Take a wire and form a triangle of the wire with the help of solder iron.

(iii) With the help of the coloured glaze paper, mark the exterior angle and the corresponding interior opposite angles.

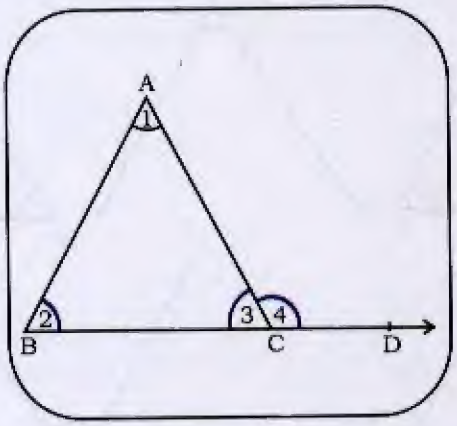


Fig. 4.1.14

Demonstration

(i) $\angle 4$ is an exterior angle of the triangle.

(ii) $\angle 1$ and $\angle 2$ are the two corresponding interior opposite angles.

(iii) $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$ can be measured with the help of a protractor.

(iv) Exterior angle is always greater than either of the interior opposite angles.

Use

- Model can be used to explain the concept of exterior angle and the corresponding interior opposite angles.
- Model can also be used to verify the result that the exterior angle of a triangle is equal to the sum of two interior opposite angles.
- The result that sum of the three angles of a triangle is 180° can also be verified experimentally using this model.

ACTIVITY 15

Concurrent Lines in Different Types of Triangles

Materials

Cardboard, saw, scale, transparent sheet, marker pen.

Preparation

(i) Draw various types of triangles such as equilateral triangle, scalene triangle and isosceles triangle on a cardboard.

(ii) Cut them neatly.

(iii) Cut four equilateral triangles of same size on transparent sheet. On one of them, draw medians and centroid (G). On second triangle, draw angle bisectors and incentre (I). On the third triangle, draw perpendicular bisectors of the sides and circumcentre (O). On fourth, draw altitudes and orthocentre (H) with the help of a marker pen.

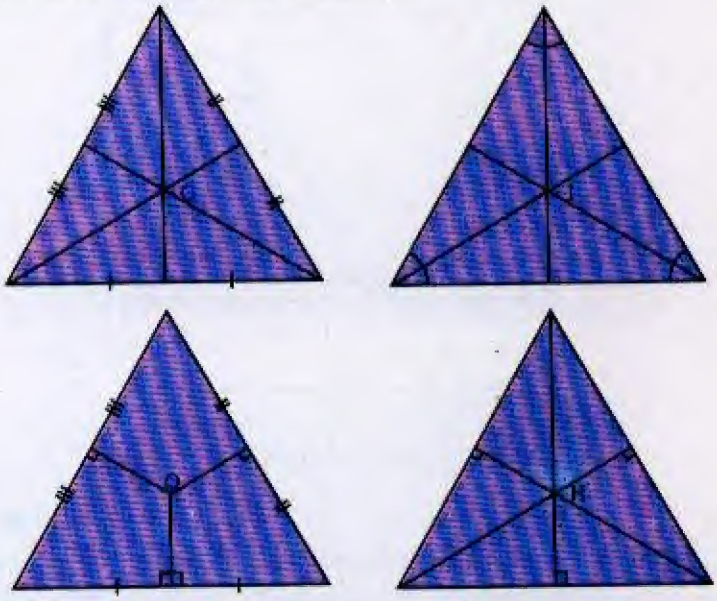


Fig. 4.1.15 (a)

(iv) Cut four isosceles triangles on transparent sheet of the size equal to that of cardboard isosceles triangle. On first, draw medians and centroid (G); on second draw angle bisectors and incentre (I). On third, draw perpendicular bisectors of

the sides and circumcentre (O). On fourth, altitudes and orthocentre (H) with the help of a marker pen.

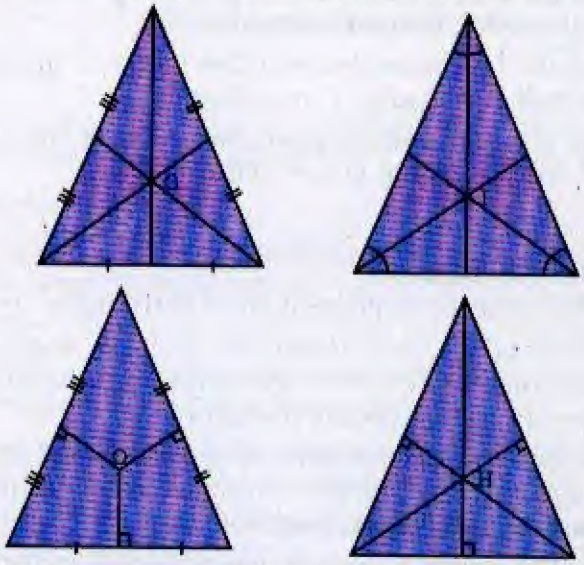


Fig. 4.1.15 (b)

(v) Similarly draw medians, angle bisectors, perpendicular bisectors of the sides and altitudes for scalene triangles on four transparent scalene triangles of same size.

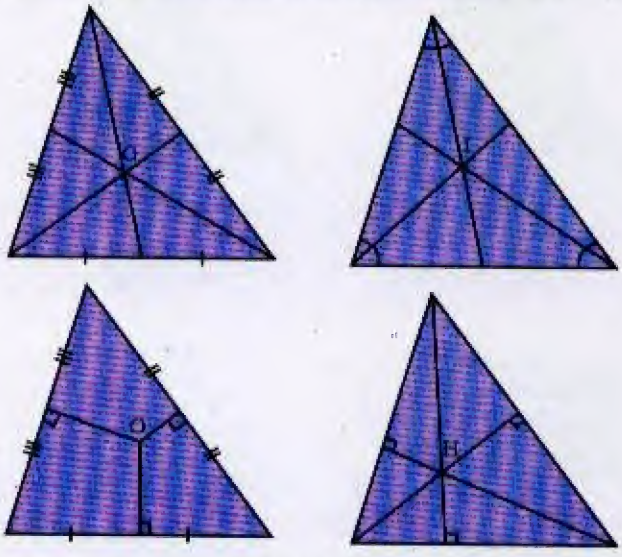


Fig. 4.1.15 (c)

Demonstration

(i) Place all the four equilateral triangles one over the other and observe that G, I, O and H coincide.

(ii) Place all the four isosceles triangles one over the other and observe that G, I, O and H are collinear.

(iii) Place all the four scalene triangles one over the other and observe that G, H and O are collinear.

Use

This model can be used

- to give the concept of different types of triangles.
- to show the point of concurrence of medians, angle bisectors perpendicular bisectors of the sides angle bisectors and altitudes of various types of triangles.
- to show that centroid, incentre, circumcentre and orthocentre of an equilateral triangle coincide by overlapping the transparent equilateral triangles.
- to show that centroid, incentre, circumcentre and orthocentre of an isosceles triangles all lie on the same line which is a median to the base.
- to show that centroid, circumcentre and orthocentre of a scalene triangle lie on the same line.

ACTIVITY 16

SSS Congruency Condition

Materials

Hard board, coloured glaze papers, transparent sheet, scale, protractor, adhesive, steel wires.

Preparation

(i) Cut two triangles with SSS congruency condition and paste them on the hard board.

(ii) Cover these two triangles with coloured glaze papers.

(iii) Draw a triangle with same congruency condition and measurement on the transparent sheet.

(iv) Cut out the three line-segments from steel wires equal to the sides of the two triangles formed in step (i).

(v) Form a pocket to put the cut out line segments and the triangle on the transparent sheet.

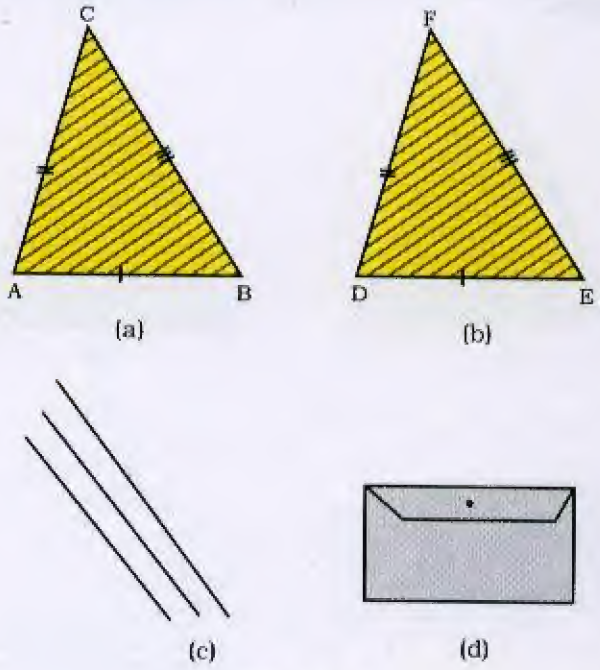


Fig. 4.1.16

Demonstration

Working is as shown below:

(i) By taking line-segments from the pocket, it can be verified that AB = DE; BC = EF and CA = FD.

(ii) By taking transparent triangle, and placing it on both the triangles, it can be verified that both the triangles ABC and DEF are congruent.

(iii) Placing the transparent triangle with other corresponding vertices, the idea of corresponding parts can be well explained.

Use

- Model can be used to show that if in any two triangles, three sides of one triangle are equal to the corresponding three sides of the other triangle, then the triangles are congruent.
- This model can also be used to explain the ideas of correspondence of vertices and corresponding parts of two triangles.

ACTIVITY 17

SAS Congruency Condition

Materials

Hard board, coloured glaze papers, transparent sheet, scale, protractor, adhesive, steel wires.

Preparation

(i) Cut two triangles with SAS congruency condition and paste them on the hard board.

(ii) Cover these two triangles with coloured glaze papers.

(iii) Draw a triangle with same congruency condition and measurement on the transparent sheet.

(iv) Cut out two line-segments from steel wires and the angle included between these line segments of the same measurement as of the two triangles formed in setp (i).

(v) Form a pocket to put the cut out line segments and the included angle; and the triangle on the transparent sheet.

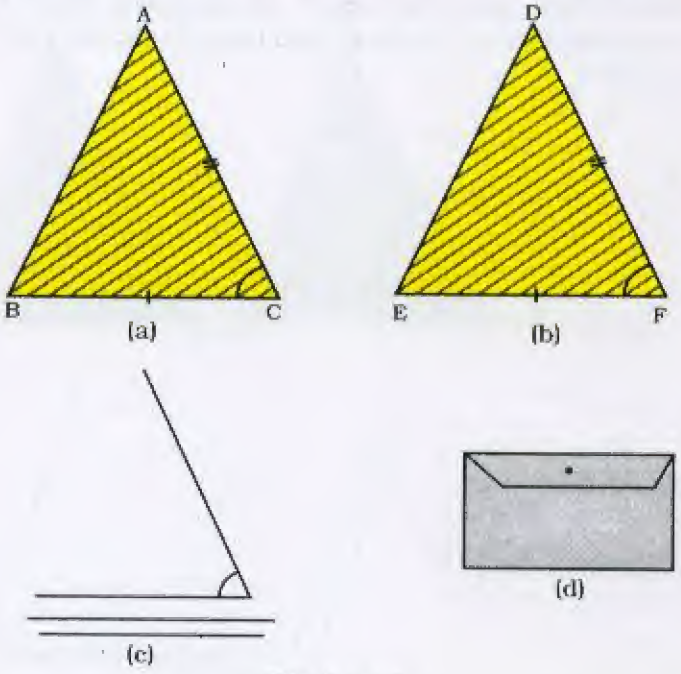


Fig. 4.1.17

Demonstration

Working is as shown below:

(i) By taking line-segments from the pocket, it can be verified that
AC = DF and BC = EF

(ii) By taking angle from the pocket, it can be verified that
$\angle C = \angle F$

(iii) By taking transparent triangle and placing it on both the triangles, it can be verified that both the triangles ABC and DEF are congruent.

(iv) Placing the transparent triangle with other corresponding vertices, the idea of corresponding parts can be well explained.

Use

- Model can be used to show that if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle, then the triangles are congruent.
- Model can also be used to explain the ideas of correspondence of vertices and corresponding parts of two triangles.

ACTIVITY 18

ASA Congruency Condition

Materials

Hard board, coloured glaze papers, transparent sheet, scale, protractor, adhesive, steel wires.

Preparation

(i) Cut two triangles with ASA congruency condition and paste them on the hard board.

(ii) Cover these two triangles with coloured glaze papers.

(iii) Draw a triangle with same congruency condition and measurement on the transparent sheet.

(iv) Cut out the equal angles from steel wires and the included line segments of the same measurement as of the two triangles formed in step (i).

(v) Form a pocket to put the cut outs of these angles and the included line segments and also the triangle on the transparent sheet.

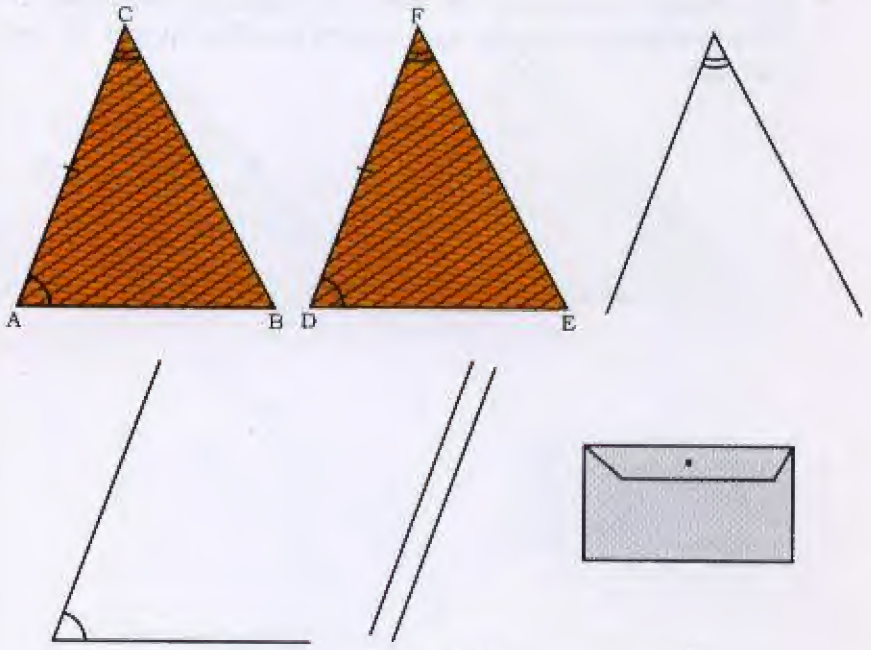


Fig. 4.1.18

Demonstration

Working is as shown below:

(i) By taking angles from the pocket, it can be verified that

$\angle C = \angle F$ and $\angle A = \angle D$

(ii) By taking line-segment from the pocket, it can be verified that

$AC = DF$

(iii) By taking transparent triangle and placing it on both the triangles, it can be verified that both the triangles ABC and DEF are congruent.

(iv) Placing the transparent triangle with other corresponding vertices, the idea of corresponding parts can be well explained.

Use

- Model can be used to show that if two angles and the included side of one triangle are equal to the two angles and the included side of the other triangle, then the two triangles are congruent.
- This model can also be used to explain the ideas of corresponding vertices and corresponding parts of two triangles.

ACTIVITY 19

RHS Congruency Condition

Materials

Hardboard/cardboard, coloured glaze papers, transparent sheet, scale, protractor, adhesive, steel wires.

Preparation

(i) Cut two triangles with RHS congruency condition and paste them on the hard board.

(ii) Cover these two triangles with coloured glaze papers.

(iii) Draw a triangle with same congruency condition and measurement on the transparent sheet.

(iv) Cut out the equal hypotenuse from steel wires, one line segment and right angle as of the two triangles formed in step (i).

(v) Form a pocket to put the cut outs of line segments and the right angle and also the cut out triangle on the transparent sheet.

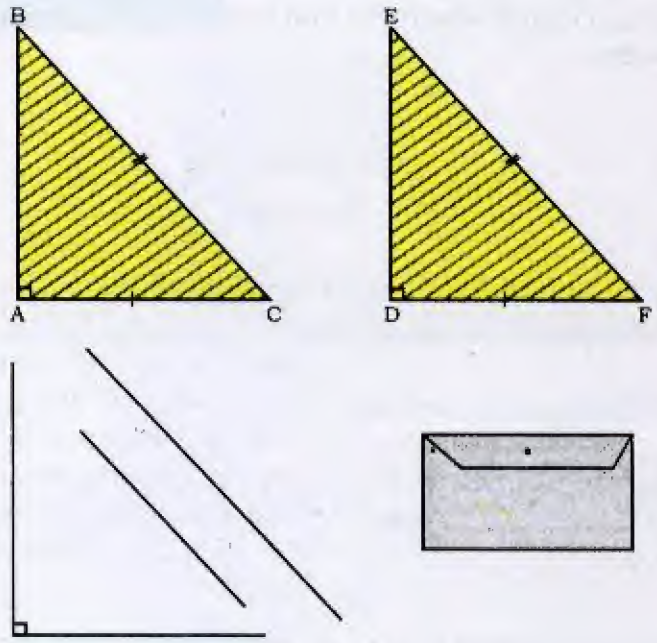


Fig. 4.1.19

Demonstration

Working is as shown below:

(i) By taking line segments from the pocket, it can be verified that CA = FD and BC = EF

(ii) By taking angle from pocket, it can be verified that :

$\angle A = \angle D$

(iii) By taking transparent triangle and placing it on both the triangles, it can be verified that both the triangles ABC and DEF are congruent.

(iv) Placing the transparent triangle with other corresponding vertices, the idea of corresponding parts can be well explained.

Use

- Model can be used to show that if in any two right triangles, hypotenuse and a side of one triangle are equal to the corresponding parts of the other triangle, then the triangles are congruent.
- This model can also be used to explain the ideas of correspondence of vertices and corresponding parts of two triangles.

ACTIVITY 20

Sum of the Angles of a Quadrilateral

Materials

Cardboard, sketch pen, adhesive, glaze paper, scale, scissors.

Preparation

(i) Take a rectangular cardboard piece.

(ii) Make a quadrilateral on the cardboard.

(iii) Make cut-outs of all the four angles of the quadrilateral with the help of a transparent sheet.

(iv) Make a pocket and put these cut outs in it.

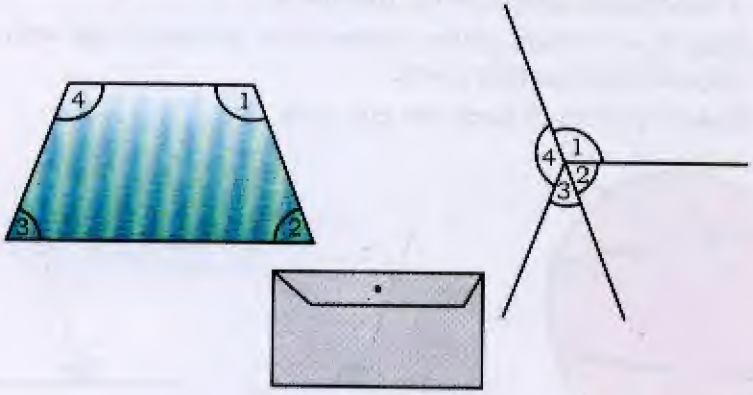


Fig. 4.1.20

Demonstration

(i) Take out the cut outs of angles from the pocket of the model.

(ii) Place the cut outs on the respective angles of the quadrilateral and show that they are equal.

(iii) Arrange the cut-outs in such a way that vertex of each angle coincide at a point (Fig. 4.1.20).

(iv) Such arrangements of cut outs show that the sum of the angles of a quadrilateral forms a complete angle and hence is equal to 360°.

Use

- Model can be used to verify the result that the sum of the angles of a quadrilateral is 360°.

ACTIVITY 21

Opposite Angles of a Cyclic Quadrilateral

Materials

Cardboard base, hard cardboard pieces, wooden pieces, scale, sketch pen, glaze paper, scissors.

Preparation

(i) Take a cardboard base.

(ii) Cover the base with glaze paper.

(iii) Cut a wooden circle and paste it on the base.

(iv) In the circle, draw a quadrilateral so that all the four vertices of the quadrilateral lie on the circle.

(v) Make the cut outs of the angles of the quadrilateral with the help of a transparent sheet.

(vi) Make a pocket to keep the cut outs.

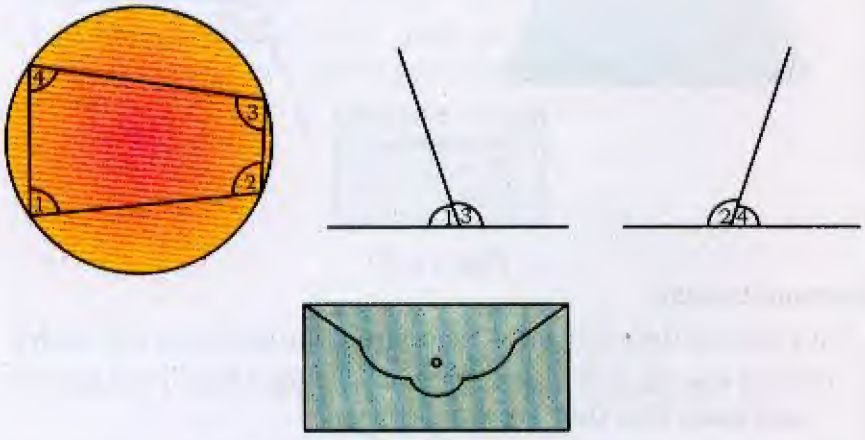


Fig. 4.1.21

Demonstration

(i) Take the cut outs from the pocket and place them on the respective opposite angles to show that they are equal.

(ii) Arrange the opposite angles in such a way that vertices of these angles coincide at a point (Fig. 4.1.21).

(iii) After arranging the opposite angles, we find that the sum of opposite angles of a cyclic quadrilateral is a straight angle or 180°.

Use

- Model can be used to verify the result that the sum of the opposite angles of a cyclic quadrilateral is 180°.

ACTIVITY 22

Area of a Trapezium

Materials

Hardboard, thermocol, coloured glaze papers, adhesive etc.

Preparation

(i) Take a piece of hardboard for the base of the model.

(ii) Cut two congruent trapezia of parallel sides a and b.

(iii) Place them on the hardboard such that they form a parallelogram as shown in the figure 4.1.22.

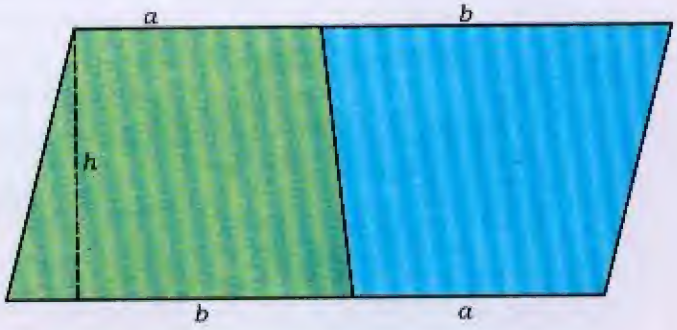


Fig. *4.1.22*

Demonstration

Arrange the congruent trapezia in such a way that they form a parallelogram as shown in figure 4.1.22.

Therefore, area of trapezium $= \frac{1}{2}$ area of parallelogram

$$= \frac{1}{2}(a + b) \times h$$

Use

- Model can be used to determine a formula for the area of a trapezium.

ACTIVITY 23

Bhaskara's proof of Pythagoras Theorem

Materials

Hardboard, thermocol, coloured glaze papers, adhesive.

Preparation

(i) Make a square of side 13 units using thermocol strip.
(ii) Make four right triangles of sides 12 units × 5 units and 13 units, i.e., $a = 12$ units, $b = 5$ units and $c = 13$ units.
(iii) Make a square of side $a - b$ i.e. 7 units.
(iv) Arrange these right triangles and square (of side 7 units) on a square of side 13 units as shown in the figure 4.1.23.

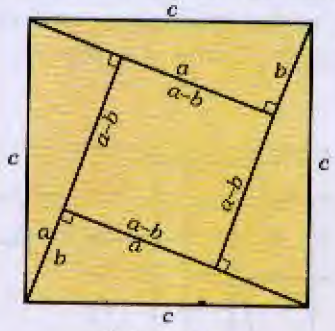


Fig. 4.1.23

Demonstration

The teacher can show that in this model a square of side c is made of four right triangles of sides a and b and a square of side $a - b$.

Therefore, c^2 = 4 (Area of right triangle) + Area of square of side $(a - b)$

$$= 4 \times \frac{1}{2} a \times b + (a-b)^2$$

$$= 2ab + a^2 + b^2 - 2ab$$

or $c^2 = a^2 + b^2$

Use

- This model can be used to verify Pythagoras Theorem.

Note

- This method is known as dissection proof for Pythagoras Theorem given by famous Indian Mathematician Bhaskara II.

ACTIVITY 24

A Proof for Pythagoras Theorem

Materials

Cardboard, thermocol, coloured glaze papers, adhesive, etc.

Preparation

(i) Make a square of side $a + b = 17$ units using thermocol strip.

(ii) Make four right triangles of sides

5 units, 12 units and 13 units, i.e., $a = 12$ units, $b = 5$ units and $c = 13$ units.

(iii) Make a square of side $c = 13$ units.

(iv) Arrange these right triangles and square (of side 13 units) on a square of side $a + b = 17$ units as shown in figure 4.1.24a.

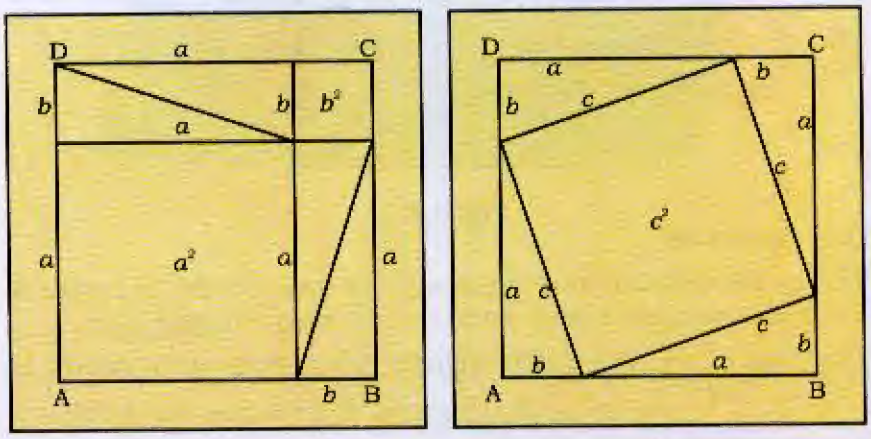


Fig. 4.1.24

Demonstration

(i) Place the four right triangles as shown in the figure 4.1.24b.

(ii) (a) Subtracting the area of the four right triangles from the square ABCD (Fig. 4.1.24a), we get the area of side c, i.e., c^2.

(b) Subtracting the area of the four right triangles from the square ABCD as shown in figure 4.1.24b, we get the area as $a^2 + b^2$.

But from (a) and (b), area must be the same.
Therefore, $c^2 = a^2 + b^2$,
which varifies the Pythagoras Theorem.

(iii) From the figure 4.1.24a, we can also write
$(a + b)^2$ = 4 × (Area of right triangle) + area of square of side c.

$$= 4 \times \frac{1}{2} ab + c^2$$

or $a^2 + 2ab + b^2 = 2ab + c^2$
i.e. $a^2 + b^2 = c^2$,
which also varifies Pythagoras Theorem.

Use

- Model can be used to verify Pythagoras Theorem as above.
- Model can also be used to verify the identity :
$(a+b)^2 = a^2 + 2ab + b^2$.

Note

- From the figure 4.1.24 (b), we can also deduce that area of square ABCD = 2 × area of rectangle of sides a and b + area of square of side a + area of squaere of side b
$= 2ab + a^2 + b^2$
$= (a + b)^2$
i.e. $(a + b)^2 = a^2 + b^2 + 2ab$

ACTIVITY 25

Tangram

Materials

Plywood board, hardboard, wooden margin, knobs, coloured paper, scissors, glue.

Preparation

(i) Cut a square of size 40cm × 40cm from the plywood board.

(ii) Make a square of size 24cm × 24cm leaving equal margin on the above plywood board.

(iii) Cut a square PQRS of size 24cm × 24cm from hardboard such that it exactly fits in the square made by margin in step (ii).

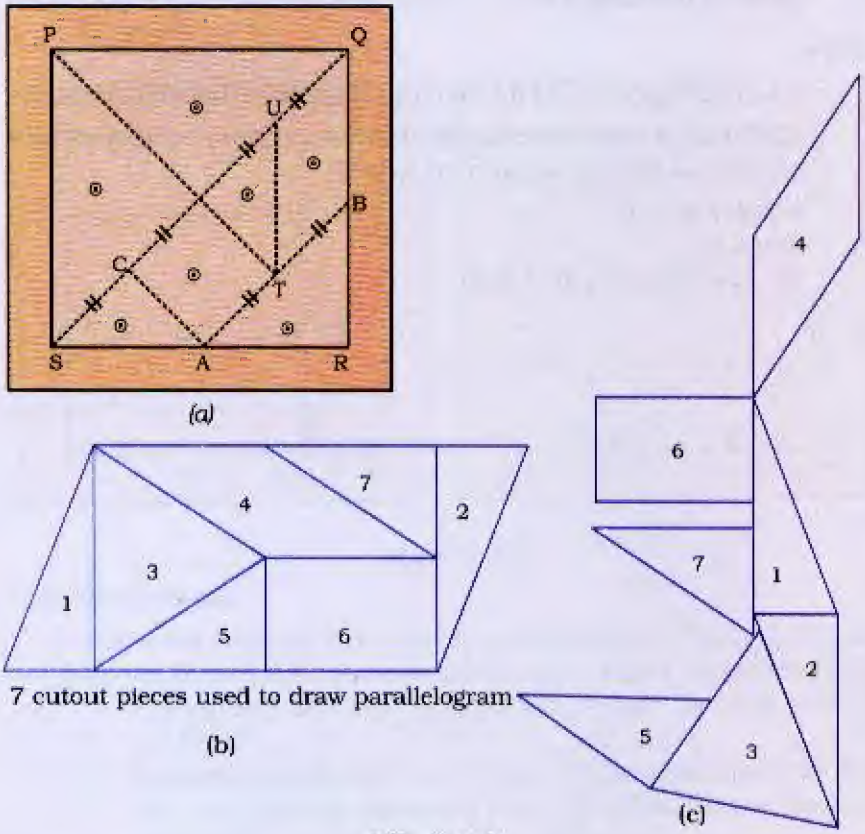


Fig. *4.1.25*

(iv) The square in step (iii) is cut into 7 pieces along dotted lines as shown in figure 4.1.25 a, with A and B as mid-points of sides SR and RQ, respectively. TU || BQ and PT ⊥ AB, where T is the mid-point of AB. Also AC ⊥ SQ.

(v) Fix knobs in various geometrical shapes obtained in step (iv), at appropriate places as shown by (.) in the figure.

Demonstration

(i) From the seven pieces shown in the model, recognition of various geometrical shapes such as triangles, squares, parallelograms, trapeziums, etc., can be taught.

(ii) Concept of congruent triangles and similar triangles can be illustrated with the help of this model.

Use

- Model can be used for the recognition of different geometric shapes and also for recreational purpose as shown in figure 4.1.25. Arrange the 7 cut out pieces on the drawing sheet in the manner as shown in the figure 4.1.25b and c. Now with the help of a pencil make the outline of the picture. Remove all the pieces and finish the drawing with good free hand.
- Various designs can be developed using tangrams.
- The students should be asked to remove all the geometrical shapes and make the margined square empty. They should be asked to refix all the shapes in the margin so as to again take the shape of a square.

Note

- This is, popularly known as famous *Chinese Puzzle*.

ACTIVITY 26

Area of a Circle

Materials

Hard wood, saw, glaze papers, scissors, adhesive, scale, pencil, eraser, sketch pens, drawing sheets, etc.

Preparation

(i) Cut a circular and a rectangular piece of wood.

(ii) Make a border on all the four sides of the rectangular piece of wood. Paste a coloured paper on the circular piece.

(iii) Cut the circle into 16 sectors of equal measurements.

(iv) Make a trace copy of these sectors and paste them on the rectangular sheet as shown in the figure. These sectors so arranged form a rectangle whose dimensions are πr and r.

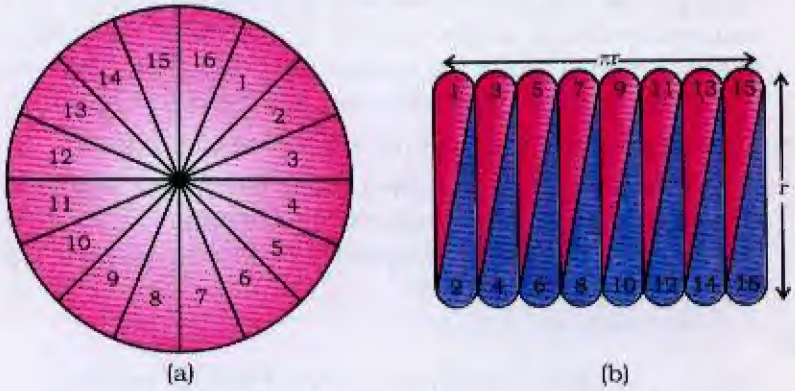


Fig. 4.1.26

Demonstration

The formula for the area of the circle can be explained as below:

Area of the rectangle = $l \times b$

Here l is $\frac{1}{2}$ circumference of the circle and $b = r$ (radius).

So $l \times b = \frac{1}{2} \times 2\pi r \times r = \pi r^2$ square units.

Note

(i) Take one sector which can be taken as a triangle with radius of the circle as approximate height of the triangle. Then

Area of triangle = $\frac{1}{2} \times r \times b$ (b = base of the triangle)

Therefore, area of circle = $16 \times \frac{1}{2} r \times b = \frac{1}{2} r(16 \times b)$

$= \frac{1}{2} r \times 2\pi r = \pi r^2$ sq units

Here, $(16 \times b)$ is equal to the circumference of the circle.

(ii) For more accurate result the number of sectors may be taken as large as possible.

Use

- Model can be used for explaining the formula of the area of a circle as shown in figure 4.1.26.

ACTIVITY 27

Net of a Cylinder

Materials

Thermocol sheet, transparent sheets, glaze paper, adhesive, scissors, scale, compasses, drawing pins, sketch pen.

Preparation

(i) Take a rectangular piece of thermocol sheet.

(ii) Cover it with coloured glaze paper.

(iii) Take a rectangular piece of transparent sheet (22 cm × 19 cm nearly).

(iv) Cut two circles from the transparent sheet with radius equal to 3 cm nearly the radius of the cylinder.

(v) Attach rectangular piece and the two circles of transparent sheet on thermocol sheet.

(vi) Two circles will become the top and bottom of the cylinder.

(vii) By folding the rectangular sheet, cylinder can be made.

(viii) Net of the cylinder is considered as top and bottom (circular in shape) and a rectangular piece of sheet.

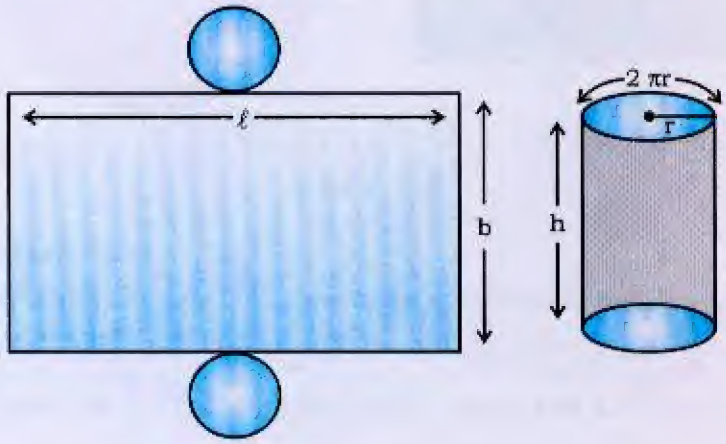


Fig. 4.1.27

Demonstration

(i) The breadth of the rectangular sheet is equal to height of the cylinder.

(ii) The length of the rectangular sheet equals $2\pi r$ which is the circumference of the circular top or bottom of the cylinder.

(iii) The Area of the rectangle = $l \times b$

$= 2\pi r \times h$

$= 2\pi rh$ = curved surface area of the cylinder

(iv) Adding $\pi r^2 + \pi r^2$ to the above, the whole surface area of the cylinder

$= 2\pi rh + 2\pi r^2$

Use

- This model can be used to explain the concept of curved surface and whole surface of a cylinder and their formulae as above.

ACTIVITY 28

Relation between the Volumes of a Cylinder and a Cone with same Base Radius and same Height

Materials

Thermocol, transparent sheet, scissors, sketch pen, scale.

Preparation

(i) With the help of a transparent sheet, make a cone with height h and radius of the base r.

(ii) With the similar transparent sheet, make a cylinder with same height 'h' and radius of base same as that of the cone (Fig. 4.1.28).

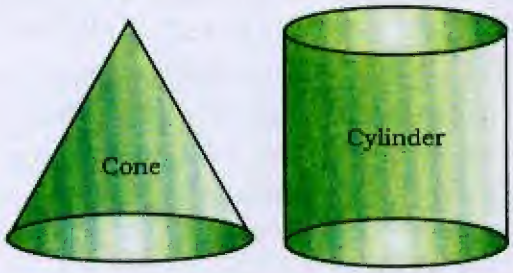


Fig. *4.1.28*

Demonstration

(i) Take some sand (or water) and fill the cone with it.

(ii) Pour the sand (or water) of the cone into the cylinder.

(iii) Again fill the cone with sand (or water) and pour into the same cylinder.

(iv) Repeat this process three times.

(v) It will be found that after filling the sand (or water) through the cone three times, the whole cylinder will be filled with sand (or water).

Use

- Model can be used to verify that volume of the cylinder = 3 (volume of the cone with same base, radius and height).

ACTIVITY 29

Some Activities by Paper Folding

Material

Hardboard, coloured paper, scale, adhesive and sketch pens, papers

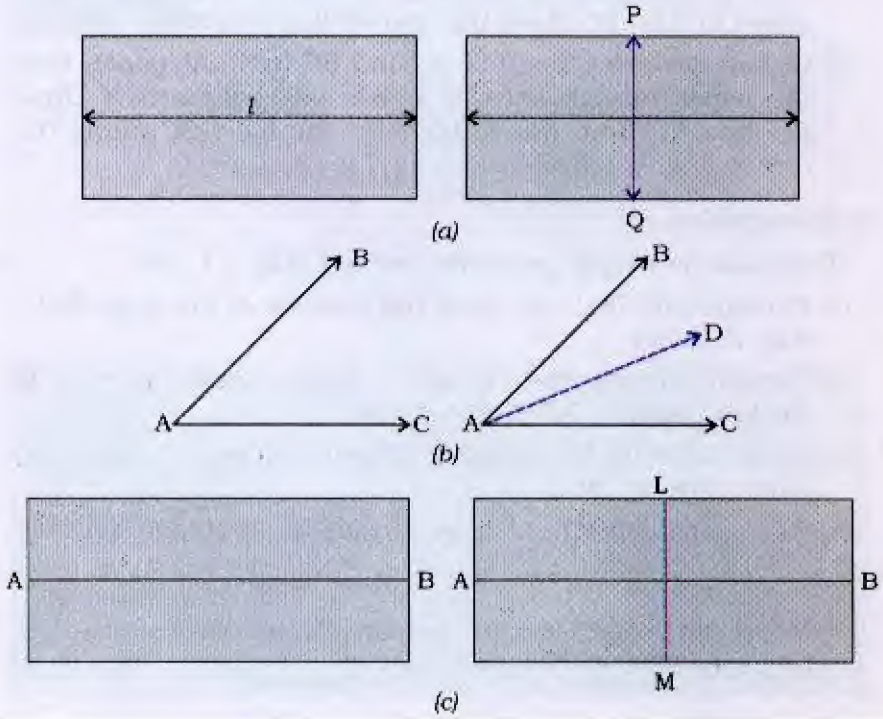
Preparation

(i) Take a hardboard of measurement 12cm × 12cm.

(ii) Take a rectangular paper and draw a line l on it. Fold the paper so that line l folds on itself. Unfold the paper and draw a line PQ along the crease (Fig. 4.1.29a).

(iii) Cut out an angle from the paper and fold it so that one ray falls on the other. Unfold the paper and draw a ray AD along the crease (Fig. 4.1.29b).

(iv) Draw a line segment on a rectangular paper. Fold the paper to join the two end points of the segment. Unfold the paper and draw a line LM along the crease (Fig. 4.1.29c).



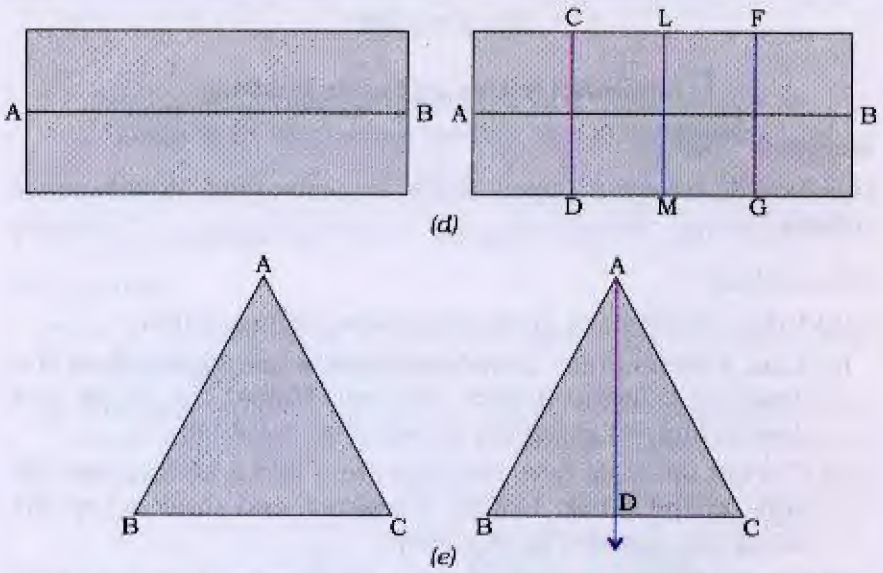


Fig. *4.1.29*

(v) Draw a line segment on rectangular paper. Join the end points twice by folding the paper twice. Unfold the paper and draw lines CD, LM, FG along the creases (Fig. 4.1.29d).

(vi) Cut an isoscles triangular region ABC from the paper. Fold this paper through A so that side BC folds along itself. Draw the line AD from the vertex A to the base BC along the crease (Fig. 4.1.29e).

Demonstration

(i) Explain that PQ is perpendicular to l (Fig. 4.1.29a)

(ii) Demonstrate that ray AD is the bisector of the angle BAC. (Fig. 4.1.29b)

(iii) Explain that the line LM is the perpendicular bisector of the line segment AB (Fig. 4.1.29c).

(iv) Explain that the line segment AB gets divided into four equal parts. (Fig. 4.1.29d)

(v) Explain that AD is the altitude of the triangle ABC (Fig. 4.1.29e)

Use

- Model can be used to demonstrate different types of activities by paper folding.

4.2 Activities at the Secondary Stage

ACTIVITY 1

Algebraic Identity : $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

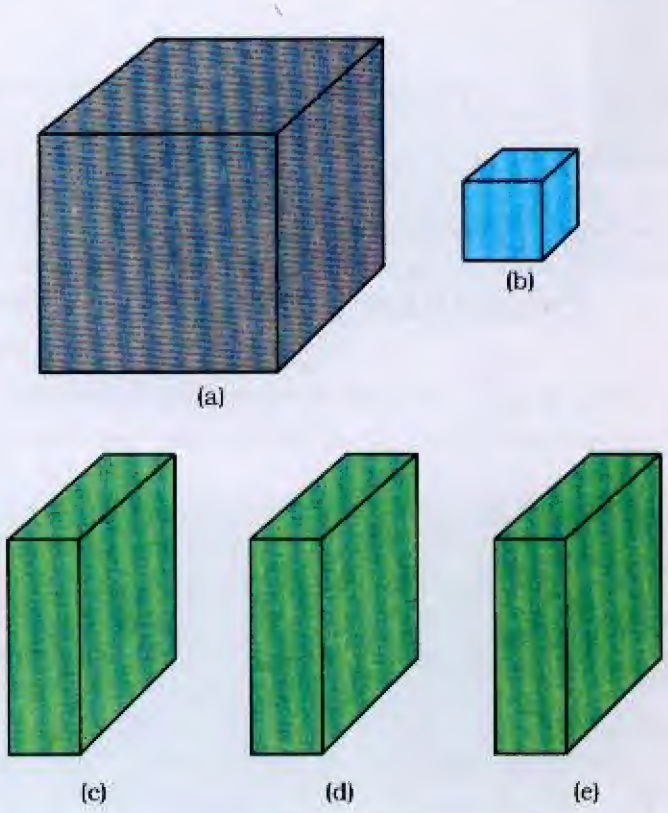
Materials

Acrylic Sheet, cutter, glaze paper, wooden board, saw, sketch pens, adhesive.

Preparation

(i) Take a cube of side $a = 3$ units and one more cube of side $b = 1$ unit as shown in figure 4.2.1a and b using wooden board.

(ii) Cut out three cuboid of dimensions $a \times a \times b$ or $3 \times 3 \times 1$ as shown in figure 4.2.1 (c), (d) and (e) using wooden board.



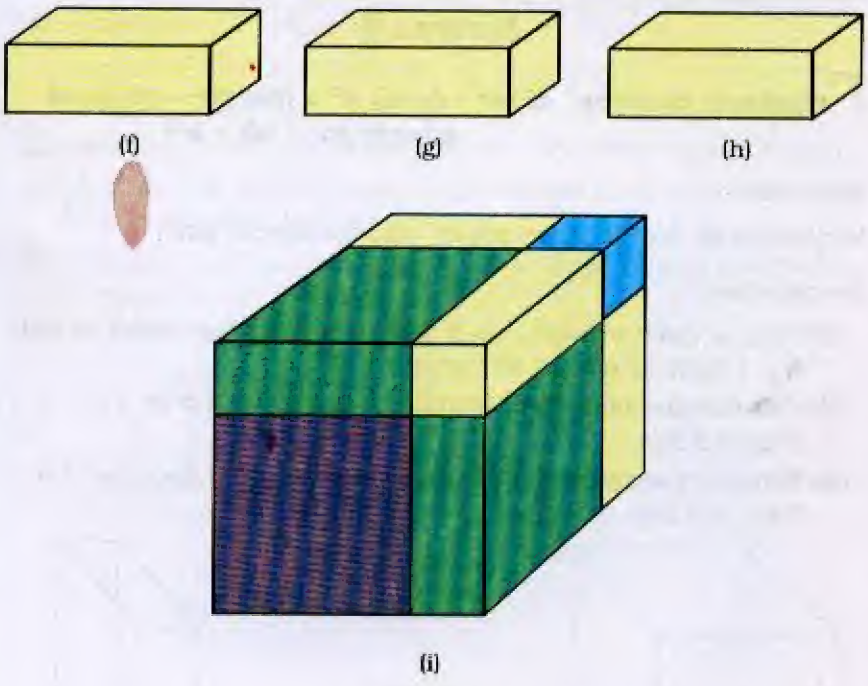


***Fig.** 4.2.1*

(iii) Cut out three cuboid of dimensions $a \times b \times b$ or $3 \times 1 \times 1$ as shown in figure 4.2.1 (f), (g) and (h) using wooden board.

Demonstration

(i) Assemble all the cuboids given in (a), (b), (c), (d), (e), (f), (g) and (h) in order as shown in figure 4.2.1 i which is a cube of side $(a + b)$.

Thus, $(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

(ii) By suitable arrangements of cubes and cuboids, identity $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$ can be verified.

Use

- Model can be used to verify algebraic identity $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

Note

- This model can also be used at the Elementry Stage.

ACTIVITY 2

Algebraic Identity: $a^3+b^3 = (a+b)\,a^2 + (a+b)b^2 - ab\,(a+b)$
$= (a+b)\,(a^2 - ab + b^2)$

Materials

Acrylic sheet, cutter, glaze paper, wooden block, saw.

Preparation

(i) Take a cube of side $a = 3$ units and another cube of side $b = 1$ unit as shown in figure 4.2.2 a and b.

(ii) Cut out the cuboid of dimensions $a \times a \times b$ or $3 \times 3 \times 1$ (Fig. 4.2.2c).

(iii) Cut out the cuboid of dimensions $a \times b \times b$ or $3 \times 1 \times 1$ (Fig. 4.2.2d).

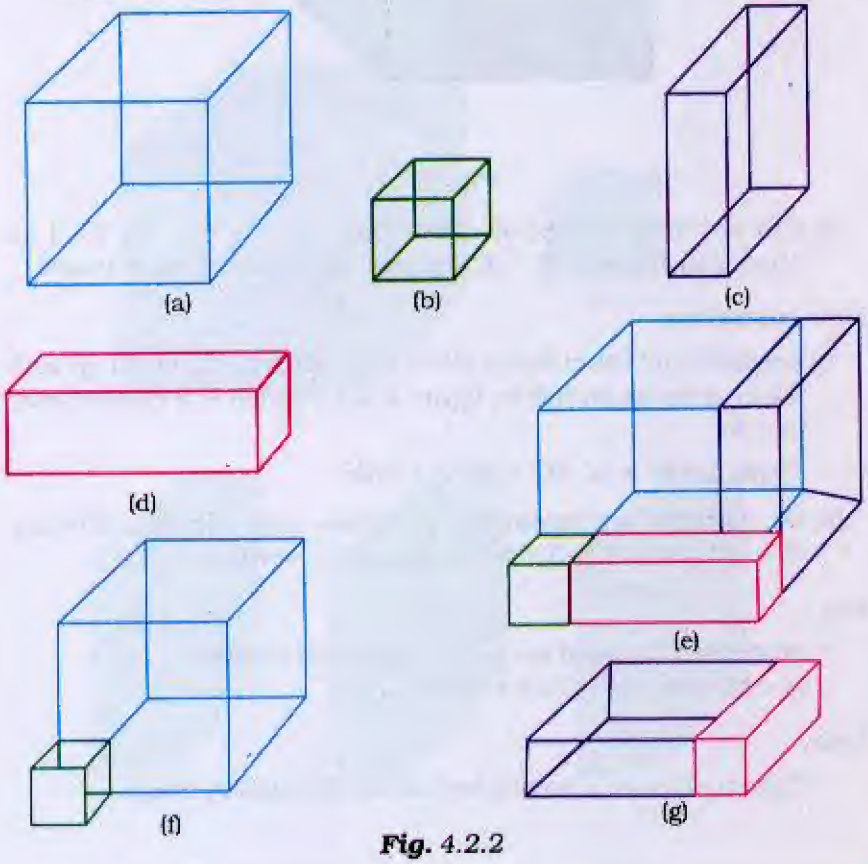


Fig. 4.2.2

Demonstration

(i) Assemble all the cuboids (a), (b), (c), (d) on the table as shown in figure 4.2.2 (e).

(ii) Remove (c) and (d), we are left with $a^3 + b^3$, in figure 4.2.2(f)

(iii) Join the two cuboids (c) and (d) as shown in the figure 4.2.2 (g). It is a cuboid of volume $(a+b)ab$

(iv) Remove $(a+b)\ ab$ from the sum of $(a+b)\ a^2$ and $(a+b)\ b^2$ to obtain $a^3 + b^3$. So that

$$a^3 + b^3 = (a+b)a^2 + (a+b)b^2 - ab\ (a+b)$$
$$= (a+b)\ (a^2 + b^2 - ab).$$

Use

- Model can be used to verify algebraic identity :
  $a^3 + b^3 = (a + b)\ (a^2 - ab + b^2)$

ACTIVITY 3

Algebraic Identity : $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Materials

Acrylic sheet, wooden board, sketch pens, glaze paper, scissors, adhesive, etc.

Preparation

(i) Take $a = 3$ units, $b = 1$ unit so that $a - b = (3 - 1)$ i.e. 2 units

(ii) Make a cuboid of size $(a - b) \times a \times a$ cubic units, using wooden board as shown in figure 4.2.3 (a)

(iii) Make another cuboid of size $(a - b) \times a \times b$ or $2 \times 3 \times 1$ cubic units, using same wooden board as shown in figure 4.2.3 (b)

(iv) Make one more cuboid of size $(a - b) \times b \times b$ or $2 \times 1 \times 1$ cubic units as shown in figure 4.2.3 (d)

(v) Make a cube of size $b \times b \times b$ or $1 \times 1 \times 1$ as shown in figure 4.2.3 (c)

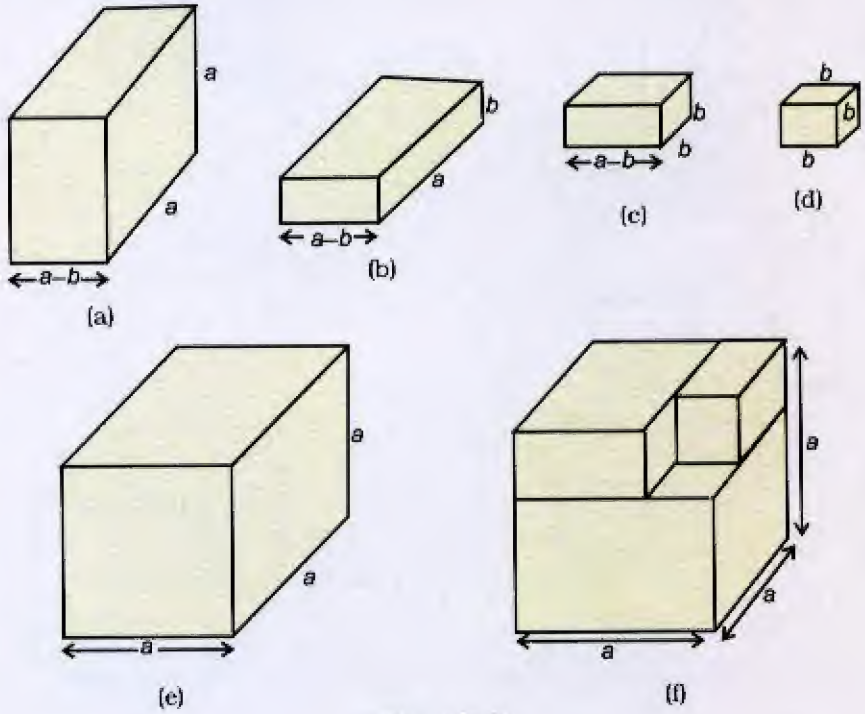


Fig. 4.2.3

(vi) Make a cube of size $a \times a \times a$ or $3 \times 3 \times 3$ cubic units from acrylic sheet to keep the model safely as shown in figure 4.2.3 (f)

Demonstration

Assemble all the three cuboids in such a way that they give a cube of size $3 \times 3 \times 3$ cubic units. By suitable arrangements of cubes and cuboids, identity $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ can be verified as follows:

Adding (a), (b), (c) and (d), we get (f) i.e.

$a^3 = (a - b) \times a \times a + (a - b) \times a \times b + (a - b) \times b \times b + b \times b \times b$
$= (a - b)(a^2 + ab + b^2) + b^3$

Subtracting b^3 from both sides, we get

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Use

- Model can be used to verify the algebraic identity:
  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Use

Model can be used to verify that:

- Areas of parallelograms on the same base and between same parallels are equal in area.
- Area of triangle and a parallelogram on the same base and between same parallels $\Rightarrow$ area of a triangle = $\frac{1}{2}$ area of the parallelogram.

ACTIVITY 10

Making an Isosceles Trapezium ABCD into an Equivalent Area of a Rectangle

Materials

Plywood board, hardboard, coloured papers, scale, adhesive, scissors, wooden margins.

Preparation

(i) Take a plywood board of size 40cm × 30cm and paste coloured paper on it.
(ii) Draw an isosceles trapezium ABCD on the plywood board in which AD = BC.
(iii) Fix wooden margins along the sides of the trapezium ABCD.
(iv) Cut a hardboard piece of size ABCD.
(v) Draw AE perpendicular to DC and cut right triangular portion ADE from the above hardboard.
(vi) Fix wooden margins BF and CF such that BF = DE and AE = CF as shown in the figure 4.2.10.

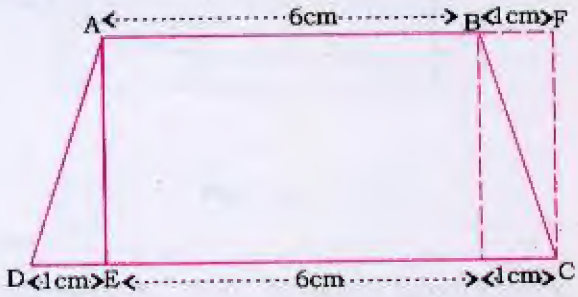


Fig. 4.2.10

Demonstration

(i) Take out the triangular hardboard ADE and keep it in such a way as to get the rectangle AECF.
(ii) Explain that the area of the trapezium ABCD is equal to the area of the rectangle AECF.

Use

- Model can be used to explain equivalence of area of rectangle with isosceles trapezium.

ACTIVITY 11

Verification of Pythagoras Theorem

Materials

Plywood board, hardboard, coloured papers, scissors, wooden margins, adhesive.

Preparation

(i) Cut a square of size 60 cm × 60 cm from the plywood board and paste a coloured glaze paper on it.

(ii) Make a right triangle with sides 24 cm, 7 cm and 25 cm of different coloured glaze paper and paste it on the centre of the board and name it as ABC.

(iii) Fix wooden margins along the sides of the triangle ABC.

(iv) Make squares on the sides AB, AC and BC in the triangle ABC using wooden margins as shown in figure 4.2.11.

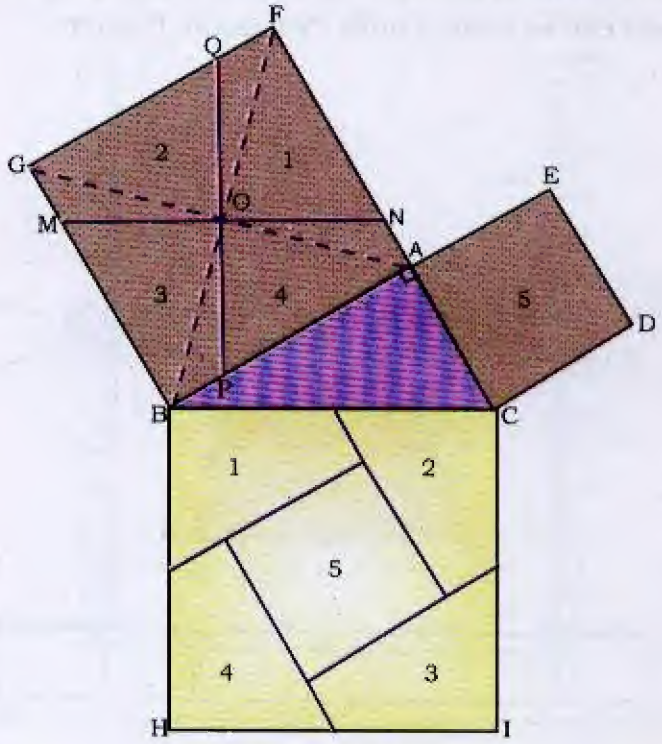


Fig. 4.2.11

(v) Cut the squares of sides AB and AC from a hardboard to exactly fit in the square made in step (iv) as shown in figure 4.2.11.

(vi) Cut the square ABGF of hardboard in four congruent parts in such a way that these four parts together with the square ACDE exactly fit in the square BCIH.

(vii) For this, follow the steps given below:

(a) Through the point of intersection O of diagonals, draw MN || BC.

(b) Construct right bisector QP of MN through O.

(c) Cut the square ABGF along the lines MN and QP.

Demonstration

Take out the four pieces of square ABGF together with the square ACDE and arrange them in the square BCIH in such a way that they exactly fit there. This verifies the Pythagoras theorem.

Use

- Model can be used to verify Pythagoras Theorem.

ACTIVITY 12

Verification of Thales Theorem

Materials

Hardboard, screws, coloured glaze papers, adhesive, scale, 4 pulleys, thread, marker, scissors, white chart paper.

Preparation

(i) Cut a piece of hardboard having a size of 40 cm × 40 cm and paste some white chart paper of same size on it.

(ii) Draw a horizontal line, 2 cm above the bottom of the paper pasted in step (i) above.

(iii) Draw two perpendicular lines, 34 cm apart on this horizontal line at the points A and B (leaving 3 cm on each side of the edge of the paper) and graduate these lines.

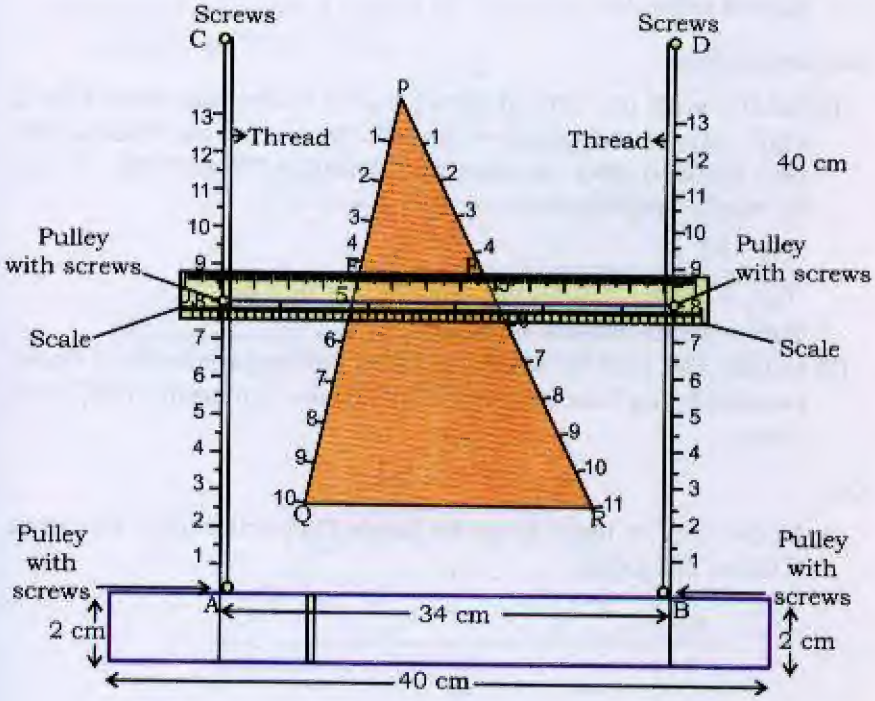


Fig. 4.2.12

(iv) Cut a triangular piece of hardboard and paste coloured glaze paper on it and place it between the perpendicular lines AC and BD such that its base is parallel to the horizontal line as drawn in figure 4.2.12.

(v) Graduate the other two sides of the triangular piece as shown in the figure 4.2.12.

(vi) Put the screws along the horizontal line 34 cm apart and two more screws on the top of the board at the point C and D (34 cm apart) such that A, B, C and D become four vertices of a rectangle.

(vii) Take a scale and make four holes on it as shown in the figure 4.2.12 and fix four pulleys at these holes with the help of screws.

(viii) Fix the scale on the board using the thread tied to nails fixed at point A, B, C, and D passing through the pulleys as shown in the figure 4.2.12 so that the scale is parallel to the horizontal line AB and can be moved up and down over the triangular piece freely.

Demonstration

(i) Set the scale on vertical lines parallel to the base line of the Δ PQR, say, at the points E and F. Measure the distance PE and EQ and also measure the distance PF and FR. It can be easily verified that

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

This verifies Thales Theorem.

(ii) Repeat the activity stated in (i) by moving scale up or down parallel to the base and verify the Thales Theorem in different cases.

Use

- Model can be used to verify Basic Proportionality Theorem (Thales Theorem).

ACTIVITY 13

Ratio of the Areas of Similar Triangles

Materials

Plywood board, hardboard, coloured glaze papers, scale, adhesive, scissors, wooden margin.

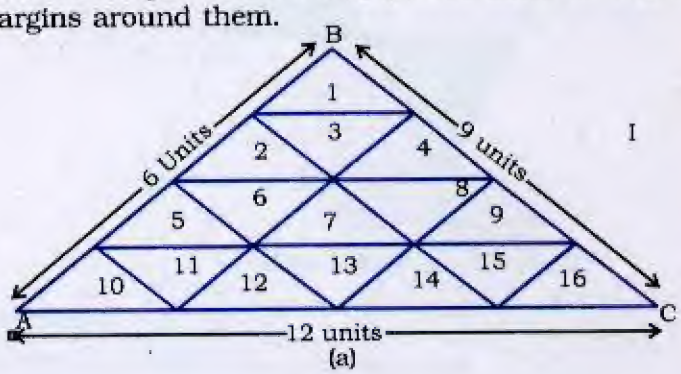
Preparation

(i) Cut a plywood of size 60 cm × 30 cm and paste coloured glaze paper on it.

(ii) Cut two triangular pieces I and II of hard board of dimensions AB = 6 units, BC = 9 units, CA = 12 units) and GE = $\frac{3}{4}$ AB, GF = $\frac{3}{4}$ BC, EF = $\frac{3}{4}$ CA so that they are similar.

(iii) Divide both the triangles into smaller similar triangles as shown in figure 4.2.13 and paste different coloured paper on each triangle.

(iv) Fix the triangles I and II on plywood taken in step (i) with margins around them.



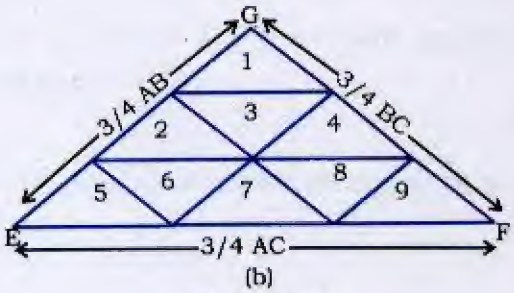


Fig. *4.2.13*

Demonstration

(i) Count the number of unit triangles in a shown in the model as 1,2,3... In this case the number of unit triangles is 16.

(ii) Count the number of unit triangles in b which is 9.

(iii) Verify that

$$\frac{\text{Area of Triangle I}}{\text{Area of Triangle II}} = \frac{16}{9} = \frac{12^2}{\left[\frac{3}{4}12\right]^2} = \frac{12^2}{9^2}$$

$$= \frac{(\text{Side})^2 \text{of Triangle I}}{(\text{Side})^2 \text{of Triangle II}} = \text{Ratio of the squares of corresponding sides of triangles.}$$

Use

- Model can be used to verify the result that ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

ACTIVITY 14

Mid-point Theorem

Material

Plywood, black colour, scale.

Preparation

(i) Cut out a Δ ABC from plywood.

(ii) Mark the mid points of AB and AC as D and E, respectively and join D to E. Paint Δ ADE with grey colour.

(iii) Cut out another triangle CEF congruent to Δ ADE. Paint this triangle with grey colour.

(iv) Place this triangle (step iii) as shown in the figure 4.2.14.

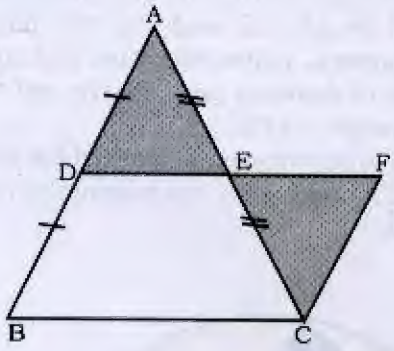


Fig. 4.2.14

Demonstration

(i) Observe that BCFD is a parallelogram.

(ii) Also DE = $\frac{1}{2}$ BC as DE = EF and opposite sides of parallelogram are equal

Use

- Model can be used to demonstrate mid point theorem.

ACTIVITY 15

Angles of a Cyclic Quadrilateral

Materials

Plyboard, cardboard, drawing pins, glaze paper, sketch pens, scissors, adhesive.

Preparation

(i) Cut out a circle of radius 7 cm on a cardboard and paste yellow glaze paper on it.

(ii) Draw a cyclic quadrilateral and extend one of its sides CD to the point E to make an exterior ∠ADE.

(iii) Use four different coloured glaze papers to represent four angles of the quadrilateral.

(iv) On a piece of cardboard, cut out three angles of measure equal to that of ∠A, ∠B and ∠C and paste blue, red and black glaze papers, respectively on the cut outs.

(v) With the help of drawing pins, fix the red coloured angle on the exterior angle ∠ADE.

(vi) With the help of drawing pins, fix the blue and black coloured angles on ∠A and ∠C, respectively inside the cyclic quadrilateral.

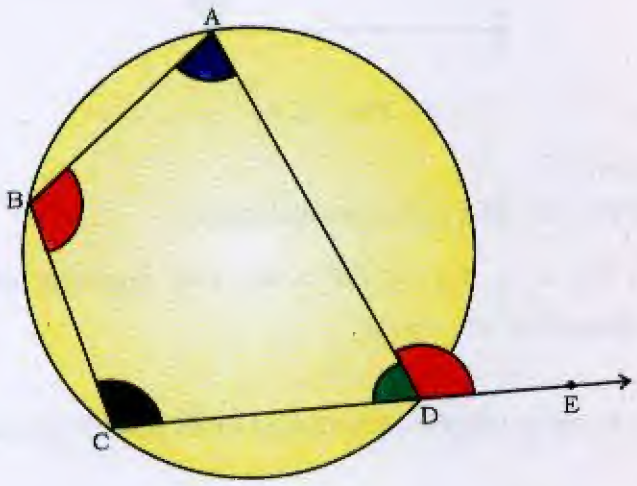


Fig. 4.2.15

Demonstration

(i) Remove the drawing pins and place the red coloured cardboard angle on ∠B to show that exterior ∠ADE is equal to its interior opposite ∠B.

(ii) Again remove the blue and black coloured cardboard angles from the cyclic quadrilateral and place them on a straight line to show that the opposite angles of a cyclic quadrilateral are supplementary.

Use

Model can be used to verify that:

- Opposite angles of a cyclic quadrilateral are supplementary.
- Exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

ACTIVITY 16

Angles in a Circle

Materials

Plyboard, cardboard, drawing pins, glaze papers, sketch pens, scissors.

Preparation

(i) Draw a circle of radius 7 cm on a cardboard sheet and paste a glaze paper on it. Let O be the centre of the circle and AD be one of its chords.

(ii) Take points B, E and C on the circle. Join AO, OD, AB, BD, AE, EC, AC and CD.

(iii) Draw a tangent PQ at the point A of the circle.

(iv) Bisect ∠AOD, by the line segment OF.

(v) On another cardboard sheet, cut out the angles of measures equal to that of ∠ABD, ∠ACD and ∠CAQ and colour them.

(vi) With the help of drawing pins, fix these cardboard angles as shown in the figure.

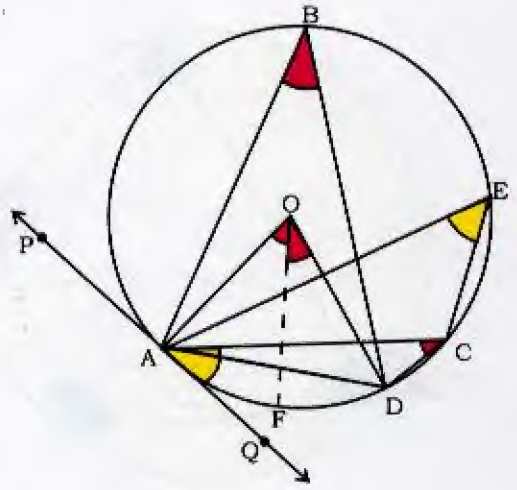


Fig. 4.2.16

Demonstration

(i) Remove the coloured cardboard ∠ABD and ∠ACD and place them one over the other. As the two angles completely cover each other, we can say that angles in the same segment of a circle are equal.

(ii) Again, remove the coloured cardboard ∠ABD and ∠ACD one by one, and place them on ∠AOD so that ∠ABD and ∠ACD are adjacent to each other. We will observe that ∠AOD is covered completely by these two angles. This shows that the angle subtended by an arc at the centre is twice the angle subtended by the same arc at any point on the remaining part of the circle.

(iii) Remove the drawing pin and place the yellow coloured cardboard angles ∠CAQ on ∠AEC. As they completely cover each other, we can say that the angle between the tangent and the chord is equal to the angle subtended by the chord in the alternate segment.

Use

- Model can be used to verify that angle between a tangent and chord of a circle is equal to the angle subtended in the alternate segment.

ACTIVITY 17

The Angles subtended by an Arc at the Centre and at any Point on the remaining Part of the Circle

Material

Cardboard, glaze paper, scissors, sketch pen, scale, adhesive, protractor, transparent sheet.

Preparation

(i) Take a rectangular cardboard.

(ii) Cover the cardboard with glaze paper and make it base for the model.

(iii) Make a circle on the base of the cardboard.

(iv) By taking an arc, make a wooden cut out of a sector. The sector will be making an angle at the centre of the circle.

(v) With the same arc, make a cut out of an angle at the remaining part of the circle.

(vi) Prepare a pocket to keep the cutouts of the angles.

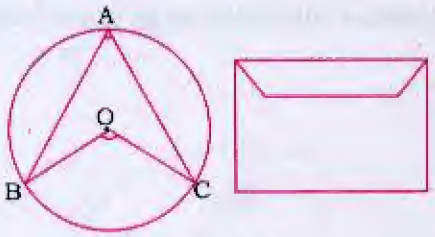


Fig. 4.2.17

Demonstration

(i) Take the cutouts from the pocket of the model.

(ii) With the help of a transparent sheet or by placing a protractor on the cutouts, explain that the angle subtended at the centre is double the angle subtended at any point on the remaining part of the circle.

Use

- Model can be used to verify that the angle subtended at the center is double the angle subtended at any point on the remaining part of the circle.

ACTIVITY 18

Chords and Tangents in a Circle

Materials

Plywood, chart paper, coloured glaze paper, nail, thread, adhesive, etc.

Preparation

(i) Take a plywood board of size 50 cm × 40 cm and fix coloured glaze paper on it.

(ii) Draw a circle of radius 10 cm on a chart paper and cut out the circular region from it.

(iii) Fix the circular region with adhesive on the plywood board.

(iv) Take any two points A and B on the circle and two other points C and D such that line joining C and D crosses the line joining A and B at a point P and fix up nail at A, B, C and D.

(v) Tie threads at A and C and extend these upto B and D, respectively and tie them at these places.

(vi) Similarly, select the points A', B' and C', D' and graduate space on paper under them.

(vii) Fix up nails at C', D' and A', B'. This results as in figure 4.2.18 (a).

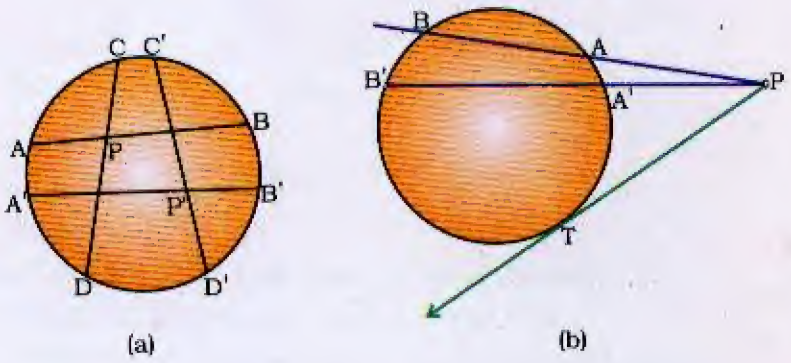


Fig. 4.2.18

(viii) Take another sheet of chart paper of size 20 cm × 12 cm.

(ix) Draw a circle of a radius 5 cm on it.

(x) Take a point P at a distance of 10 cm from the centre of the circle.

(xi) Draw tangent P T to the circle from the point P. Also secants PAB and PA'B' passing through the point P.

(xii) Fix up nails at points P, A, B, A', B' and T.

(xiii) Stretch threads along PAB, PA' B' and PT and graduate the space on the paper under them. This results in figure 4.2.18 (b).

Demonstration

(i) In figure 4.2.18 (a), take the measurements of PA and PB, also of PC and PD.

(ii) Verify that PA × PB = PC × PD.

(iii) Similarly, verify that P'A' × P'B' = P'C' × P'D'

(iv) In figure 4.2.18 (b) measure the lengths of PT, PA, PB and PA' and PB'. Verify that (Fig. 4.2.18b): $PT^2 = PA \times PB$.

Also $PT^2 = PA' \times PB'$

Use

- Model can be used to verify some results relating to intersecting chords, secants and tangents of a circle.

ACTIVITY 19

Formation of Cones from Sectors of a Circle

Materials

Thermocol sheet, transparent sheets, glaze paper, sketch pen, white chart paper, nails, black marker pen.

Preparation

(i) Take a thermocol base of size 19 cm × 22 cm.

(ii) Paste a coloured glaze paper on this base.

(iii) Paste a margin of a different colour on all the four sides of the base.

(iv) Cut out two circles of radius 10 cm from a transparent sheet.

(v) From one of the circles, cut out a sector of degree measure 120°.

(vi) Bring together both the radii of the sector to form a cone and paste the ends.

(vii) Fix the cone on the thermocol.

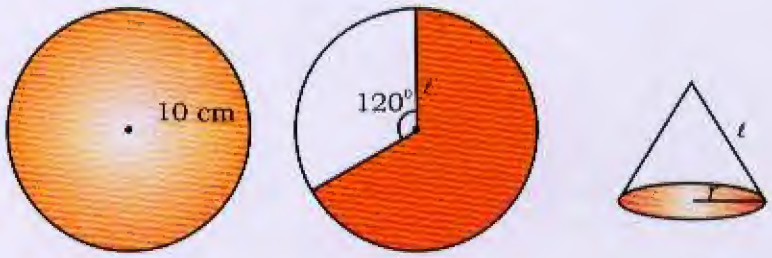


Fig. 4.2.19

Demonstration

(i) Curved surface area of the cone = Area of the sector.

(ii) Slant height (l) of the cone = radius of the circle

(iii) Circumference of the base of cone = arc length of the sector.

(iv) Radius of the base of the cone = r

(v) Measure the radius of the circle. Call it l. Then measure the radius of the base of the cone which has been fixed separately. Call it r.

(vi) Area of the sector =

$$\frac{\text{arc length}}{\text{circumference of the circle}} \times \text{area of the circle}$$

$$= \frac{2\pi r}{2\pi l} \times \pi l^2 = \pi r l$$

Use

- Model can be used to explain the formula for curved surface area of a cone.

ACTIVITY 20

Finding Volume Relationships among Right Circular Cone, A Hemisphere and A Right Circular Cylinder of the Same Radii and Heights

Materials

Cardboard, plastic sheet, cutter, plastic ball, adhesive, marker, sand or salt.

Preparation

(i) Take a plastic ball of radius, say 20 cm and cut this ball into two halves so as to obtain a hemisphere.

(ii) Take a plastic sheet and put it on the cardboard. Cut the sheet and paste it to get a right circular cone of radius 20cm and height 20cm.

(iii) Similarly, cut a plastic sheet and make a right circular cylinder of radius 20 cm and height 20 cm.

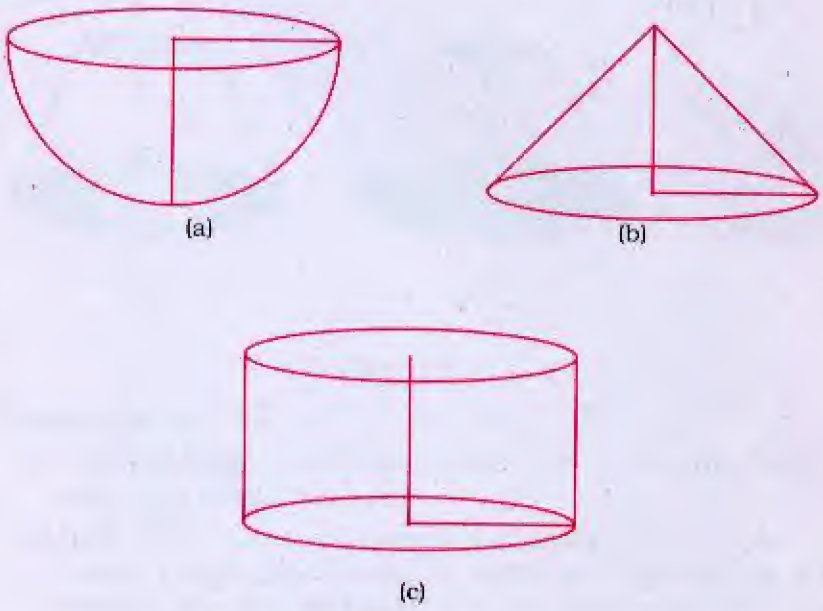


Fig. 4.2.20

Demonstration

(i) Fill the cone with salt or sand and pour it twice into hemisphere. Show that hemisphere is filled with the sand or salt.

(ii) Fill the cone with salt or sand and pour it 3 times into the cylinder. Show that cylinder is filled.

(iii) Explain that the volume relationship of cone, hemisphere and cylinder of the same height and radii are in the ratio of 1: 2: 3, respectively.

(iv) Explain the validity of the observations mathematically as follows:

Volume of the cone = $\frac{1}{3}\pi r^2 h$

Volume of the hemisphere = $\frac{2\pi}{3} r^3$

Volume of the cylinder = $\pi r^2 h$

In case, if $r = h$, then the volume relationship is

$= \frac{1}{3}\pi r^3 : \frac{2\pi}{3} r^3 : \pi r^3$

$= 1 : 2 : 3$

ACTIVITY 21

Frustum of a Cone

Materials

Acrylic sheet, scale, marker, pencil, geometry box, sketch pens, adhesive, cellotape, scissors, hardboard, saw.

Preparation

(i) Cut out a hard board of measurements 25 cm × 25 cm.

(ii) Cut out a circle of radius 10 cm from the acrylic sheet.

(iii) Cut out a sector of 120° from the circle.

(iv) Make a cone from a major sector. Let r_1 be its base radius and h_1 its height.

(v) Cut off a smaller cone of height h_2 units from this cone to get a frustum of a cone [Fig. 4.2.21 (b)]

(vi) The remaining shape can be joined to form a frustum.

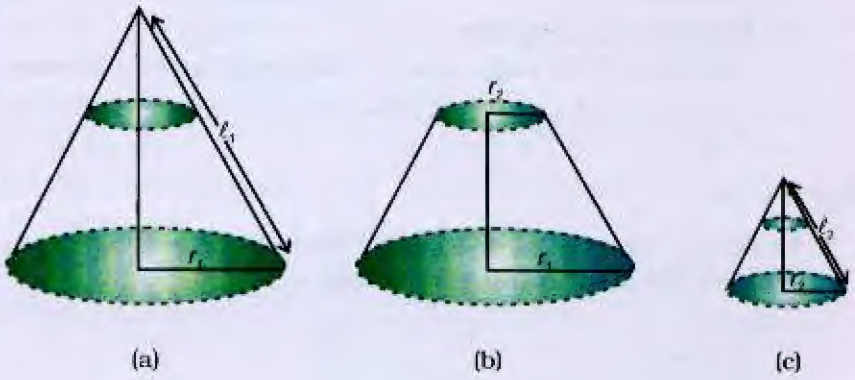


Fig. *4.2.21*

Demonstration

(i) Take the bigger cone whose radius is r_1 units, slant height l_1 units and height is h_1 units.

(ii) In the other, we cut off a smaller cone of radius r_2 units, slant height l_2 units and height h_2 units. So when we cut off a smaller cone with its radius parallel to the base of the bigger cone, we get a frustum of the cone.

(iii) We know that

(a) Curved surface area of the bigger cone
$= \pi r_1 l_1$ sq. units.

(b) Total surface area of the bigger cone
$= \pi r_1 l_1 + \pi r_1^2$ sq. units.

(c) Volume of the cone $= \frac{1}{3} \pi r_1^2 h_1$ cu. units.

(d) Curved surface area of smaller cone $= \pi r_2 l_2$ sq. units.

(e) Total surface area of the smaller cone $= \pi r_2 l_2 + \pi r_2^2$ sq. units.

(f) Volume of the smaller cone $= \frac{1}{3} \pi r_2^2 h_2$ cu. units.

(iv) Put the cut-out cone on the frustum making the bigger cone. We get:

a) The curved surface area of the frustum
= curved surface area of the bigger cone–curved surface area of the cut off cone.
$= (\pi r_1 l_1 - \pi r_2 l_2)$ sq. units.

b) Total surface area of the frustum $= \pi r_1 \ell_1 - \pi r_2 \ell_2 + \pi r_1^2 + \pi r_2^2$ sq. units.

c) Volume of the frustum
= Volume of the bigger cone – Volume of the smaller cone
$= \frac{1}{3} \pi r_1^2 h_1 - \frac{1}{3} \pi r_2^2 h_2$

Use

- Model can be used to explain the formulae for surface area and volume of the frustum of a cone.

ACTIVITY 22

Surface Area of a Right Circular Cone

Materials

Cardboard, coloured paper, marker, adhesive.

Preparation

(i) Take a sheet of cardboard of size 30 cm × 30 cm

(ii) Cut a right circular cone of radius r = 10 cm with vertex O and height OO′= h = 20 cm.

(iii) Take a point P on the circular edge so that OP is slant height say l.

(iv) Measure l and draw a circle with centre O″ on coloured paper such that O″ P′ =l i.e. radius of the circle.

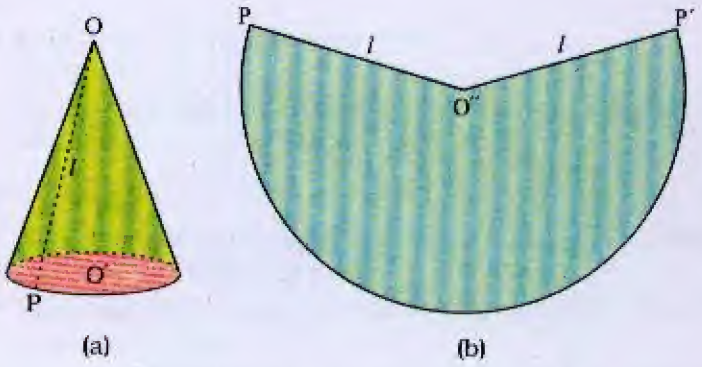
(v) Cut out the sheet of paper along the circle. What we get is a circular disc of radius l and centre O″.

(vi) Cut the disc along O″ P′.

(vii) Place the end O″ P′ of the disc along OP with O″ at O and P′ at P.

(viii) Wrap the circular disc around the cone with centre O″ fixed at O.

(ix) After coming back to OP, cut off the remaining part of the disc.



(a) (b)

Fig. 4.2.22

Demonstration

(i) Remove the wrapped portion of the disc and spread it on the plane surface.

(ii) This is a sector of radius l.

(iii) The length of the arc of this sector is just the circumference of the base of the cone i.e. $2\pi r$.

(iv) Curved surface of the right circular cone

= Area of the sector

$$= \frac{1}{2} \times (\text{arc length}) \times \text{radius}$$

$$= \frac{1}{2} \times 2\pi r \times l$$

$$= \pi rl$$

Use

- Model can be used for demonstrating surface area of a right circular cone.

ACTIVITY 23

Different Type of Prisms

Materials

Acrylic sheet, cellotape, scissors, geometry box.

Preparation

(i) Take acrylic sheet and cut out two regular pentagons, each of side 6 cm so as to form top and base of the prism.

(ii) From the acrylic sheet cut out five rectangles of dimensions 6 cm × 15 cm.

(iii) Join the breadths of each of the rectangle with one side of both the pentagons.

(iv) Join all the five rectangles length wise with the help of cellotape.

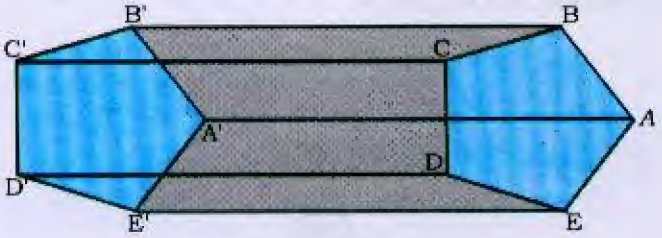


Fig. 4.2.23

Demonstration

We can demonstrate the model in order to find out the number of vertices (V), edges (E) and faces (F).

Use

- Model can be used to verify that:
  $V - E + F = 2$
- Model can also be used to explain surface area and volume of prisms.

Note

- By using two equilateral triangles in place of pentagons, we can obtain triangular prism.
- By using two hexagons in place of pentagons, we can obtain hexagonal prism.
- By using squares and rectangles in place of pentagons, we can obtain square prisms and rectangular prisms.

ACTIVITY 24

Different Type of Pyramids

Materials

Acrylic sheet, cello tape, scissors, geometry box, etc.

Preparation

(i) From the acrylic sheet cut out equilateral triangle of any side, say 6 cm.

(ii) From acrylic sheet cut out 3 isosceles triangles with base equal to the side of the equilateral triangle and base angle, say 75°.

(iii) Join the base of the triangles with the sides of the equilateral triangle with cellotape both inside and outside.

(iv) Join all the sides of the isosceles triangles lengthwise with the help of the cellotape.

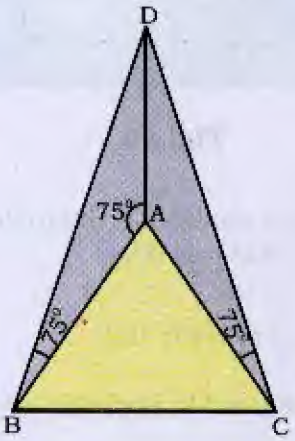


Fig. 4.2.24

Demonstration

(i) Count the number of vertices (V), edges (E) and faces (F).

(ii) Verify that V – E + F = 2.

Use

- Model can be used for verifying the formula V– E + F = 2
- Model can also be used to explain surface area and volume of pyramids.

Note

- By replacing an equilateral triangle by a regular pentagon and regular hexogen, we can obtain pentagonal and hexagonal pyramids respectively.

Activity 25

Polyhedrons and their Nets

Materials

Transparent sheet, scissors, adhesive, scale.

Preparation

(i) Draw the nets of Tetrahedron, Cube, cuboid, octahedron, Dodecahedron, Icosahedron on transparent sheet as shown below:

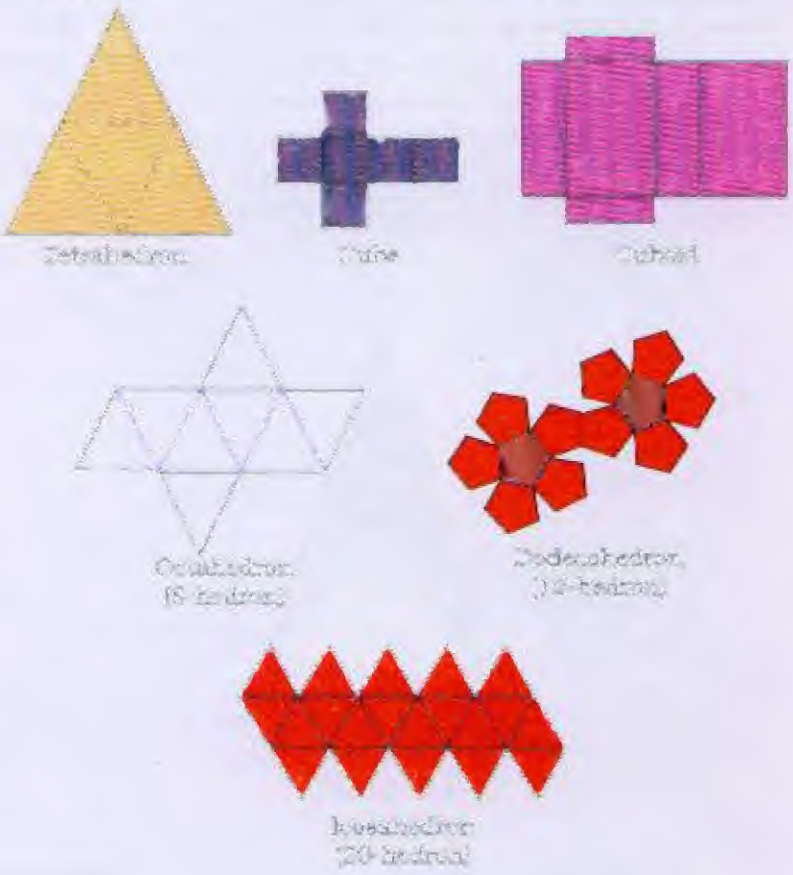


Fig. 4.2.25

(ii) Cut them along the lines and join them with the help of adhesive to get different types of polyhedron.

Demonstration

(i) Bulbs along the first column represent domain.

(ii) Bulbs along the second column represent co-domain.

(iii) Ordered pairs (A, P), (B,R) etc. represent elements of a relation.

(iv) A to H and P to U glowing bulbs represent into function.

(v) Any bulb in the second column (except V) is called the image, which defines the inverse element.

Use

The activity can be used to explain :

- Relations
- Functions

Suggestions

- If more bulbs are used in the domain, the activity can be made more effective.

Note

- In the preparation of this model, help of an electrician may be taken.

ACTIVITY 2

Quadratic Function with the help of Linear Functions

Materials

Wooden sheet, wire.

Preparation

(i) Take two wires of equal length.

(ii) Fix them at a point in the plane at right angles to each other to represent x-axis and y-axis.

(iii) Take another two pieces of wire to represent linear functions $y = x - 1$ and $y = x - 2$.

(iv) Find the point of intersection of these two linear functions with the axes.

(v) Fix one wire at points A(0, –1) and B(1,0) and the other at C(0, – 2) and D (2,0).

(vi) Fix the parabolic shaped wire represented by $y = (x - 1)(x - 2)$ in such a way that it passes through (1,0) and (2,0).

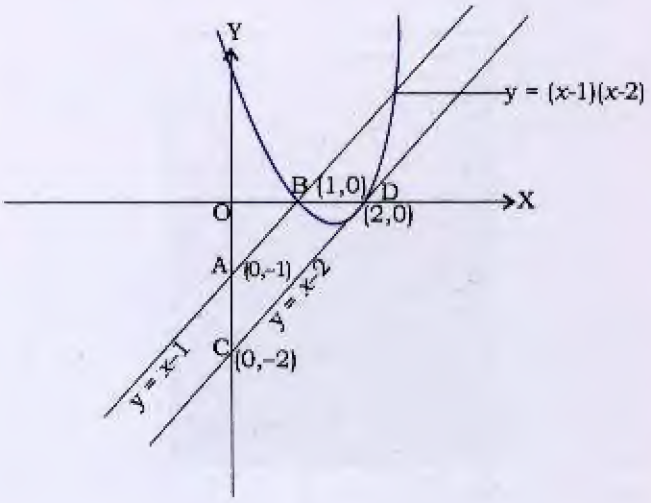


Fig. 4.3.2

Demonstration

(i) The straight line AB is representing the linear function $y = x-1$, cutting the x and y axes at (1,0) and (0, –1), respectively.

(ii) The straight line CD is representing the linear function $y = x - 2$, cutting the x and y axes at (2,0) and (0, – 2).

(iii) The product of linear functions is $y = (x - 1)(x-2)$ i.e., $y = x^2 - 3x + 2$, which represents a quadratic function. It cuts the x –axis at the same points (1,0) and (2,0).

(iv) The roots of the quadratic equation $x^2 - 3x + 2 = 0$ corresponds to the point of intersection of graph of function $f(x)$ with the x-axis.

Use

- Model can be used to explain quadratic function with the help of product of two linear functions.

ACTIVITY 3

Arithmetic Progression and Its Sum

Materials

Plastic strips, chart papers, thermocol sheets, adhesive.

Preparation

(i) Take a thermocol sheet in the shape of the rectangle ABCD.

(ii) Take some plastic strips each of equal fixed length denoted by a and some plastic strips, each of equal fixed length denoted by b.

(iii) Arrange and paste both types of strips so as to get terms a, $a + b$, $a + 2b$, ... $a + 9b$ placed at unit distance apart and arrange along the rectangle as shown in the figure 4.3.3.

(iv) The last strip ends in F along BC, extend F to C by a fixed length a so as to cut rectangle ABCD.

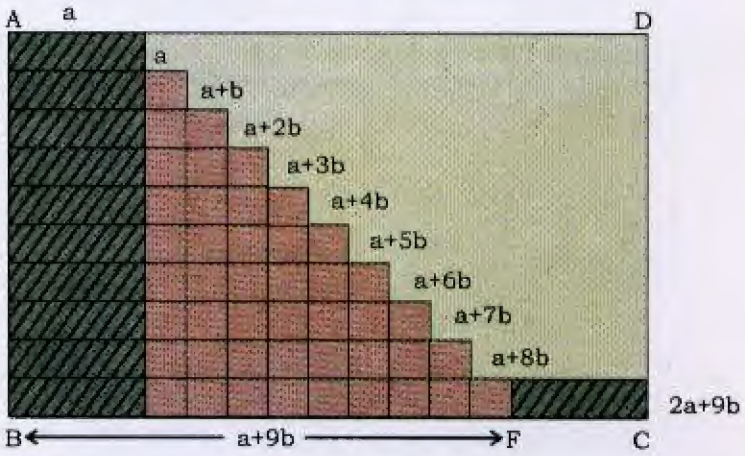


Fig. 4.3.3

Demonstration

(i) The first strip is of length a.

(ii) Second strip is of length $a + b$.

(iii) Third strip is of length $a + 2b$.

(iv) Tenth strip is of length $2a + 9b$.

(v) Strips arranged look like a stair case.

(vi) The sum of above arithmetic progression

$= a + (a+ b) + (a+2\ b) + ... + (a + 9\ b)$

$= 10a + 45\ b$

$= \frac{10}{2}\ (2a + 9\ b) = \frac{1}{2}\ [10 \times (2a + 9\ b)]$

$= \frac{1}{2}$ (Area of the rectangle ABCD where length BC $= 2a + 9\ b$ and breadth is 10 units)

Use

Model can be used to demonstrate:

- concept of arithmetic progression,
- sum of arithmetic progression.

Note

- If the arithmetic progression is a, $a+ b$, $a+ 2\ b$, ..., $a + (n–1)\ b$, then the sum of its first n terms

  $= \frac{n}{2}\ [2a + (n–1)\ b] = \frac{1}{2}\ n\ [\ 2a + (n–1)\ b]$

  = Half of the area of the rectangle whose length is $2a + (n–1)\ b$ and breadth is n-units.

ACTIVITY 4

Sum of Odd Numbers i.e. $\Sigma(2n-1)=n^2$

Material

Thermocol sheet, thermocol balls, pins, pencil, scale, adhesive, chart paper.

Preparation

(i) Take a square piece of the thermocol.

(ii) Fix chart paper on the thermocol.

(iii) Draw horizontal and vertical lines with pencil to mark squares.

(iv) Take a pin and fix a thermocol ball in it and fix it in the corner of the square

(v) Repeat the same with thermocol balls and pins on the whole board as shown in the figure 4.3.4.

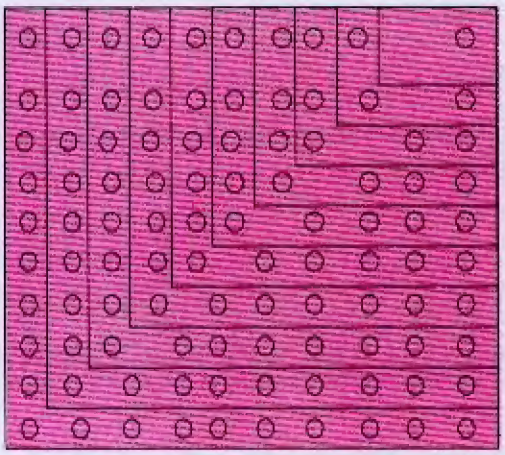


Fig. 4.3.4

Demonstration

(i) For $n = 1$, $= \Sigma(2n-1) = (2 \times 1 - 1) = 1 = 1^2$ i.e. the square on the top of the right in the figure having one ball. It can be explained as 1^2.

(ii) For $n = 2 = \Sigma(2n-1) = (2 \times 2 - 1) + (2 \times 1 - 1) = 3 + 1 = 2^2$, i.e., the next square having $1 + 3 = 4$ balls, can be explained as 2^2.

(iii) For $n = 3$, $\Sigma(2n-1) = (2 \times 3 - 1) + (2 \times 2 - 1) + (2 \times 2 - 1) = 5 + 3 + 1 = 9 = 3^2$ i.e. the next square having 9 balls can be explained as 3^2.

(iv) Taking the sum of all these cases, we get

$1+3+5+\ldots+(2n-1) = n^2$.

(v) The number n^2 on the right hand side of the result suggests the perfect square shape of configuration. In the present activity $n = 10$.

Use

- Model can be used to explain the sum of odd numbers by taking different values of n.

Suggestion

- The model can also be prepared from wooden board and nails.

ACTIVITY 5

The Sum of Square of the First n Natural Numbers, i.e.,

$$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

Material

Wooden cubes of size $(1 \times 1 \times 1)$cu unit, adhesive, nails.

Preparation

(i) Take one wooden cube of size $(1 \times 1 \times 1)$ cu cm (Fig. 4.3.5a).

(ii) Take four wooden cubes and fix them together (Fig. 4.3.5b).

(iii) Take nine wooden cubes, and fix them together (Fig. 4.3.5c).

(iv) Take sixteen wooden cubes and fix them together (Fig. 4.3.5d).

(v) Arrange all the blocks to form an echelon type of structure as shown in the figure 4.3.5 (e).

(vi) Make six such echelon type of pieces.

(vii) Arrange the six pieces to form a bigger cuboidal block and verify that its dimensions are $4 \times 5 \times 9$ cu cm.

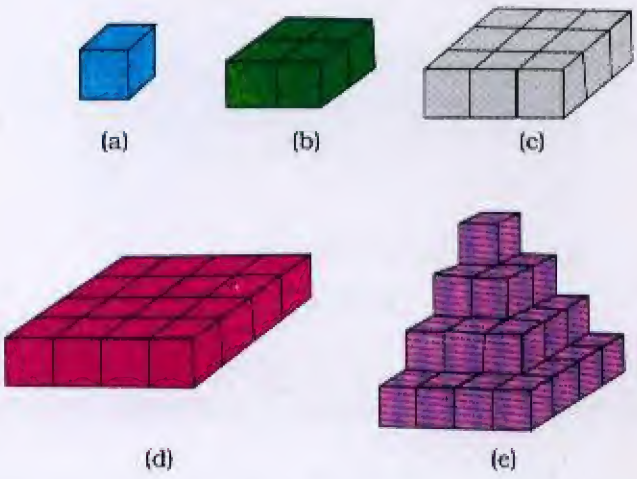


Fig. 4.3.5

Demonstration

(i) One block (Fig. 5e) represents volume $1\times(1^2+2^2+3^2+4^2)$ cu cm.

(ii) Total volume of six blocks is $6\,(1^2+2^2+3^2+4^2) = 4(4+1)\,(2\times4+1)$.

(iii) From this we can visually infer the result.

$$1^2+2^2+3^2+4^2+\ldots+n^2 = \frac{1}{6}\,n\,(n+1)\,(2n+1)$$

Use

- Model can be used to explain the sum of squares of first n natural numbers.

ACTIVITY 6

Graphs of $\sin x$ and $\sin^{-1}x$

Materials

Wooden strips, wire, soldering wire

Preparation

(i) Fix two wooden strips perpendicular to each other and intersecting at O to represent x – axis and y – axis.

(ii) Draw the graph of $\sin x$ with the values given below:

x	$-\pi$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	π
$\sin x$	0	-1	$-\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$	1	0

(iii) Take up three wires in the shape of $\sin x$, $\sin^{-1} x$ and $y = x$ and fix them on the wooden strip.

(iv) Paint the wires with different colours.

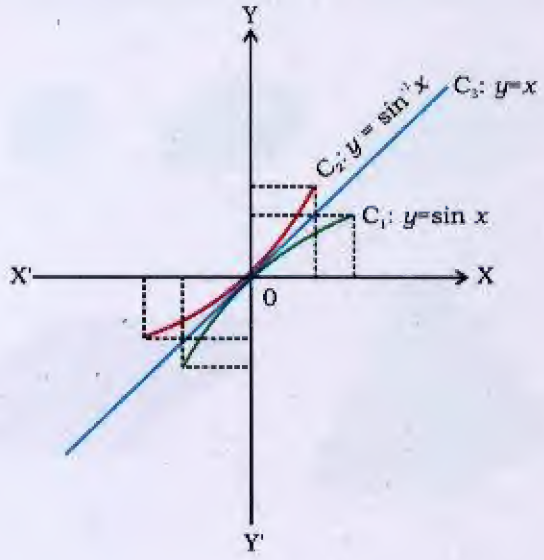


Fig. 4.3.6

Demonstration

(i) sin x crosses x-axis at the points

……:.. , $(2\pi, 0)$, $(-\pi, 0)$, $(0,0)$, $(\pi, 0)$, $(2\pi, 0)$,……

$\sin^{-1} x$ crosses y-axis at the points

(ii) ……, $(0, -2\pi)$, $(0, -\pi)$, $(0,0)$, $(0, \pi)$, $(0, 2\pi)$, ….., and $y = x$ passes through the origin.

(iii) C_1 wire represents the sin x, C_2 wire represents the $\sin^{-1} x$ and C_3 wire represents the $y = x$.

(iv) It may be noted that the sin x and $\sin^{-1} x$ are symmetrical with respect to $y = x$.

Use

- Model can be used to explain the relationship between graphs of sin x and $\sin^{-1} x$.

ACTIVITY 7

Conic Sections

Materials

Transparent sheet, scissors, hardboard, adhesive

Preparation

(i) Cut a transparent sheet in the shape of sector of a circle.

(ii) Form a right circular cone by folding the transparent sheet and using adhesive.

(iii) Fix the cone on hardboard.

(iv) Take the sections of the cone by transparent plane sheet in different positions with respect to axis and generators to obtain circle, ellipse, parabola and hyperbola (if plane cuts both parts of the cone) (Fig. 4.3.7a).

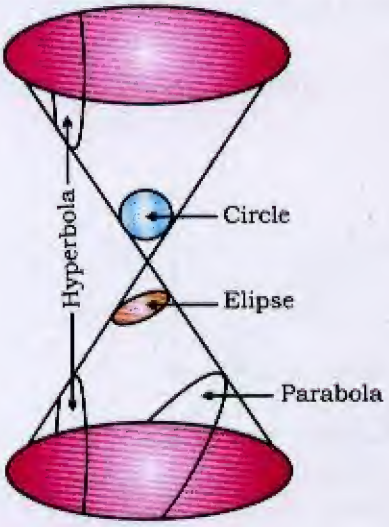


Fig. 4.3.7 (a)

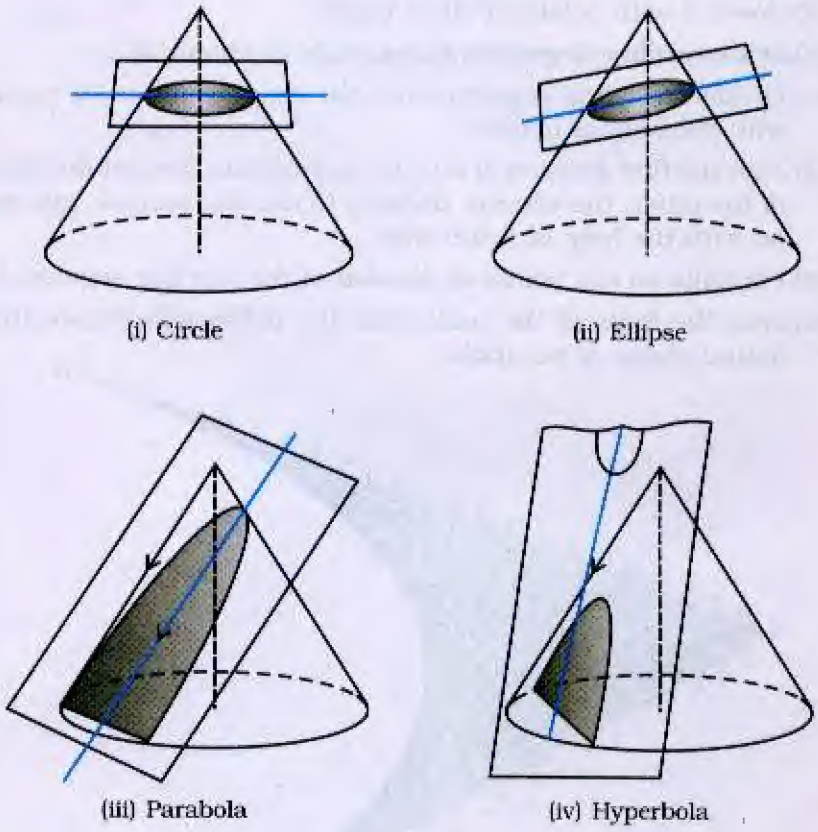
Demonstration

(i) Take a plane sheet and cut the cone in such a way that it is perpendicular to the axis of the cone, then the section will be a circle (Fig. 4.3.7b i).

(ii) Place the plane such that it is inclined slightly to the axis. The section will be an ellipse (Fig. 4.3.7 b ii).

(iii) Place the plane such that it is parallel to a generator. The section will be a parabola (Fig. 4.3.7 b iii).

(iv) Take a double right circular cone and place the plane in such a way that it is parallel to the axis and cuts the upper part as well as the lower part of the double cone. The section will be hyperbola consisting of two branches of a single curve (Fig. 4.3.7 b iv).



(i) Circle (ii) Ellipse

(iii) Parabola (iv) Hyperbola

Fig. *4.3.7 (b)*

Use

- Model can be used to explain different type of conic sections.

ACTIVITY 8

Construction of a Parabola

Materials

Plywood sheet, coloured chart paper, nails, nylon wire, marker, scale, pencil

Preparation

(i) Take a plywood sheet of size 25cm x 25cm.

(ii) Cover it with coloured chart paper.

(iii) Draw two line segments at an angle of about 50°.

(iv) Divide each line segment into the same number of parts with the help of marker.

(v) Join the first division of one line segment to the last division of the other, the second division to the last second, and so on with the help of nylon wire.

(vi) Fix nails on the points of division of the two line segments.

(vii) Join the foots of the nails with the nylon wire to see the actual shape of parabola.

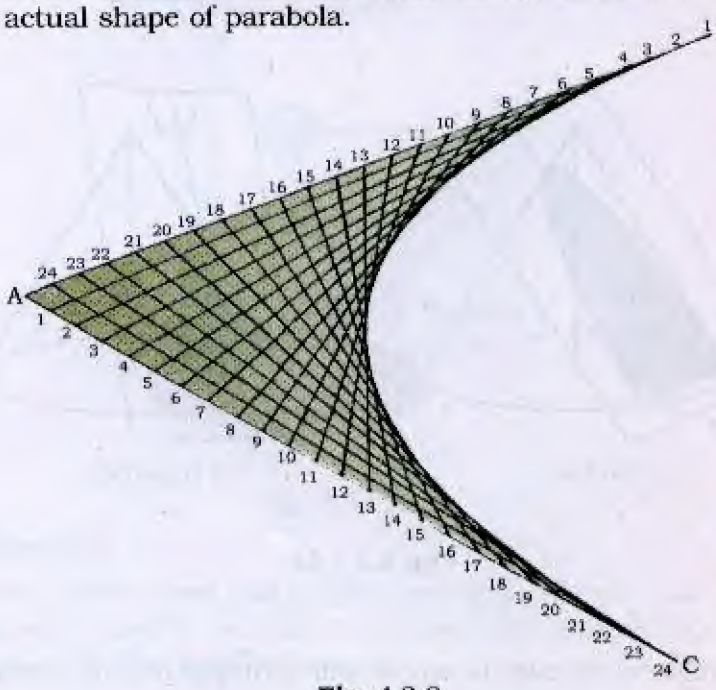


Fig. 4.3.8

Demonstration

(i) AB and AC are two line segments inclined at an angle 50°.

(ii) AB and AC are divided in 24 equal parts. Along AB, the points of division are numbered from 1 to 24 where as along AC they are numbered from 24 to 1 as shown in the figure 4.3.8.

(iii) The equally marked points of division are joined together. For example, 1 is joined with 1,2 with 2, ..., 24 with 24. The shape of a parabola is formed.

Use

- Model can be used to explain the construction of a parabola.

ACTIVITY 9

Construction of an Ellipse when Major and Minor Axes are given

Materials

Hardboard, chart paper, nylon wire.

Preparation

(i) Take a rectangular sheet of hardboard and cover it with chart paper.

(ii) Draw two concentric circles with their radii equal to the semi major and the semi minor axes of the required ellipse.

(iii) Draw radii at intervals of 10° or 15°.

(iv) Draw a vertical line from a point at which each radius cuts the outer circle and meet another horizontal line drawn from a point where the same radius cuts the inner circle.

(v) Fix the nails at the point of intersection of vertical and horizontal lines drawn parallel to minor and major axes.

(vi) Join foot of the nails by a nylon wire to get the shape of an ellipse.

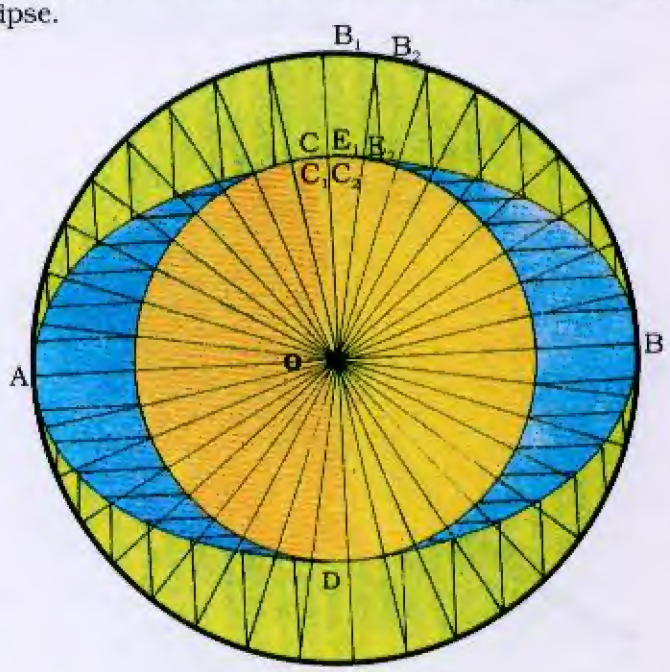


Fig. 4.3.9

Demonstration

(i) AB represents the major axis and CD represents the minor axis of the given ellipse.

(ii) OB_1, OB_2, ... represent radii of the bigger circle.

(iii) OC_1, OC_2, ... represent radii of the smaller circle.

(iv) E_1, E_2, ... are the points of intersection of the horizontal and vertical lines.

(v) The path of the points E_1, E_2, ... represents the required ellipse.

Use

- Model can be used to explain the construction of an ellipse with given major and minor axes.

ACTIVITY 10

Construction of an Ellipse

Material

Rectangular cardboard, coloured chart paper, sketch pen, scale, adhesive.

Preparation

(i) Take a rectangular card board and cover it with chart paper.

(ii) Take a rectangle ABCD on the one corner of the board.

(iii) Divide BC into 11 parts and join each of them to A.

(iv) Divide DC also into 11 parts.

(v) Draw a series of lines from X through each of the division of DC. In turn, the first meeting the first line from A to BC at A_{11}; the second one to the second one and so on.

(vi) Fix nails at the points of contact and join them with nylon wire to make part of an ellipse. Repeat this process to complete the ellipse.

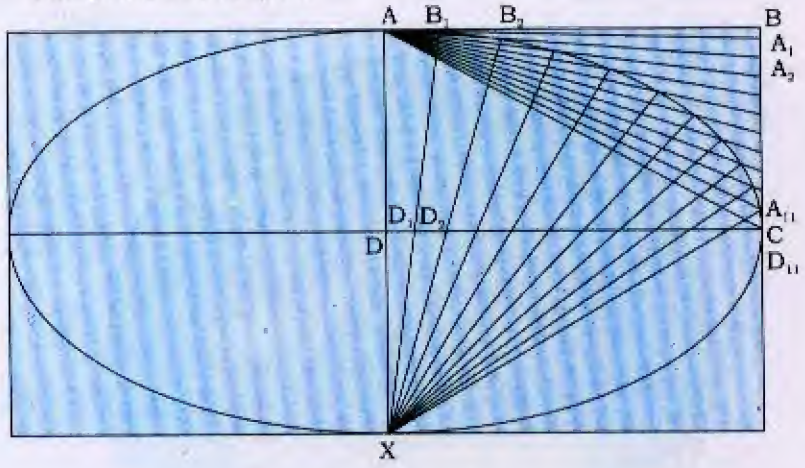


Fig. 4.3.10

Demonstration

(i) ABCD represents the rectangle.

(ii) $A_1, A_2, \ldots, A_{11}$ are the points of division of BC joined with A.

(iii) $D_1, D_2, \ldots, D_{11}$ are the points of division of DC.

(iv) XD_1, XD_2, ..., XD_{11} are the family of lines meeting the lines drawn from A to A_1,A_2, ..., A_{11}.

(v) B_1, B_2,.... B_{11} are the points of contact. Fixing nails at these points and joining them with nylon wire one can get the part of an ellipse. Repeating this process in the other three quadrant, one can construct entire ellipse.

Use

- Model can be used to demonstrate the construction of an ellipse.

ACTIVSITY 11

Construction of Parabola when Distance between Directrix and Focus is Given

Material

Hardboard, chart paper, hammer, nails, nylon wire.

Preparation

(i) Take a rectangular sheet of hardboard and cover it with chart paper.

(ii) Draw horizontal line CD as axis on the chart paper and mark focus F on CD. Also draw a vertical line AB through C to denote the directrix.

(iii) Bisect CF in V to denote the vertex.

(iv) Mark number of points P_1, P_2, ..., P_7 on VF as shown in the figure 4.3.11.

(v) Take F as a centre and radius equal to CP_1, draw arcs cutting the perpendiculars through P_1 at A_1 and A'_1.

(vi) Repeat the process and locate points A_2, A_2' and A_3, A'_3 etc. on both sides of the axis CD.

(vii) Fix nails on these points and join foot of the nails by a nylon wire to get the shape of a parabola.

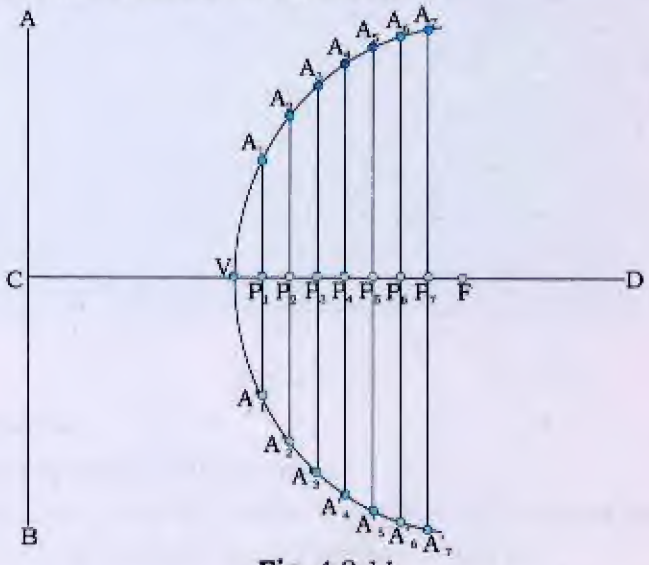


Fig. 4.3.11

Demonstration

(i) CD denote the x-axis

(ii) AB denotes the directrix

(iii) V,F denote the vertex and focus, respectively.

(iv) $A_1, A'_1, A_2, A'_2; \ldots, A_7, A'_7$ represent the position of nails.

Use

- Model can be used to explain that how a parabola can be traced with a given distance between directrix and the focus.

ACTIVITY 12

Visualising Ellipse by Paper Folding

Material

Hardboard, sketch pen, paper sheet.

Preparation

(i) Place the paper on the hardboard, cut a circle of radius 20 cm.

(ii) Mark a point F inside the circle but not at the centre.

(iii) Make a fold so that a point F′ on the circumference gets mapped onto F. (Fig. 4.3.12).

(iv) Take a point P in such a way that the line SF′ passes through it.

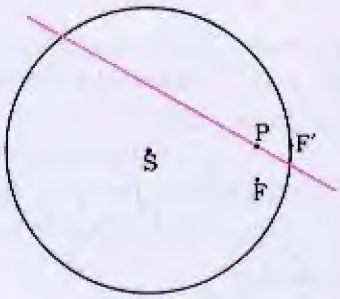


Fig. 4.3.12

Demonstration

(i) Explain that PF = PF′

(ii) SP + PF = constant (equal to the radius of the circle)

(iii) Explain that by varying F′, the point P will describe an ellipse with S and F as foci.

(iv) Explain that the fold will be a tangent to the ellipse.

(v) Explain that making a large number of folds (same F, but changing F′), we get a large number of tangents and we can see the shape of ellipse better as the more folds we make.

Use

- Activity will be suited for a geometrical construction program on the computer.

ACTIVITY 13

Pascal's Triangle

Materials

Thermocol sheets, chart papers, adhesive, match sticks.

Preparation

(i) Take a piece of thermocol sheet and cover it with chart paper.

(ii) Take match sticks and fix on the chart paper with the help of adhesive so as to make Pascal's Triangle (Fig. 4.3.6).

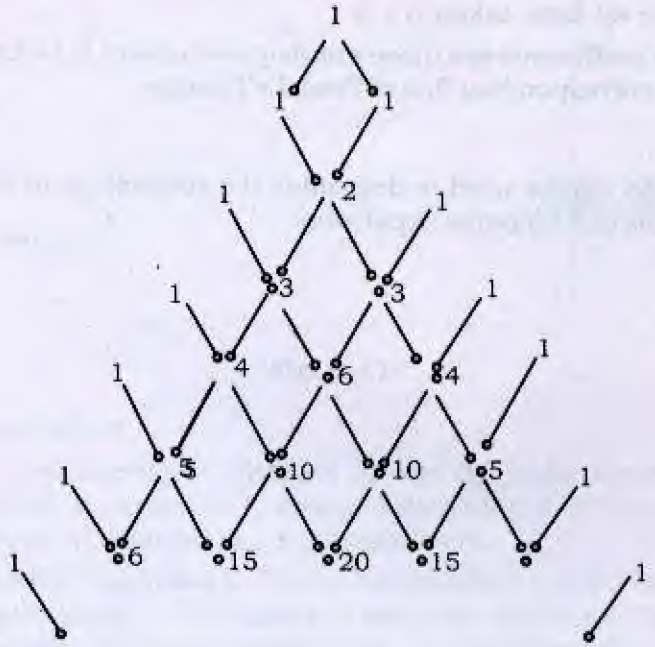


Fig. 4.3.13

Demonstration

(i) At apex of the Pascal's Triangle is 1.

(ii) Each of the lines, which follow, begins (and ends) with 1.All other numbers in a line are the sum of the above two numbers immediately to the left and right in the line.

(iii) The number series made by each diagonal line are 1 1 1 1 etc.; 1 2 3 4 etc.

(iv) Compare each row of the Pascal's Triangle with the expansion of $(a + b)^n$

Power	**Expansion**
$(a + b)^0$	1
$(a + b)^1$	$1a + 1\ b$
$(a + b)^2$	$1a^2 + 2ab + 1b^2$
$(a + b)^3$	$1a^3 + 3a^2\ b + 3ab^2 + 1b^3$
$(a + b)^4$	$1a^4 + 4\ a^3\ b + 6a^2b^2 + 4ab^3 + 1\ b^4$
$(a + b)^5$	$1a^5 + 5\ a^4b + 10a^3b^2 + 10\ a^2b^3 + 5ab^4 + 1b^5$

Here we have taken $n = 5$

(v) The coefficients are those numbers, which are to be found on the corresponding line of Pascal's Triangle.

Use

- Model can be used to determine the coefficients of different terms of a binomial expansion.

ACTIVITY 14

Application of Series Connection of Switches to Mathematical Logic

Materials

Switches, electric wire, battery and lamp.

Preparation

(i) Take two switches S_1 and S_2, first connect these in series as shown in the figure 4.3.14.

(ii) Connect Battery and Lamp so as to complete the circuit.

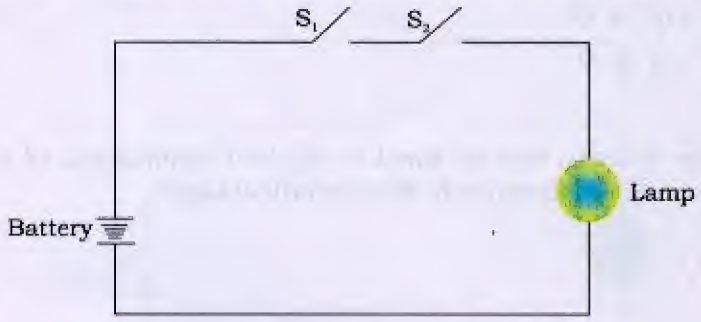


***Fig.** 4.3.14*

Demonstration

(i) Corresponding to switches S_1 and S_2, make the following logical arrangements. Assign statements p, p'; q, q' to the status of switches S_1, S_2, respectively.

(ii) Use the statement 1, 1′ to the status of the Lamp. Symbolize equivalently to the status of switches, Lamp (or the flow of current) by binary digits 0 and 1 as given below:

$p \equiv$ Switch S_1 is closed $\equiv 1$

$p' \equiv$ Switch S_1 is open $\equiv 0$

$q \equiv$ Switch S_2 is closed $\equiv 1$

$q' \equiv$ Switch S_2 is open $\equiv 0$

$l \equiv$ Lamp is on or the current flows $\equiv 1$

$l' \equiv$ Lamp is off or the current does not flow $\equiv 0$

$0' \equiv 1$ and $1' \equiv 0$

(iii) Following table may be used for demonstrating status of switches and Lamp's *on* or *off* positions:

Status of Switches S_1	S_2	*Denotes series switch operation*	*Status of Lamp*
0	0	0.0	0 → Lamp is off
0	1	0.1	0 → Lamp is off
1	0	1.0	0 → Lamp is off
1	1	1.1	1 → Lamp is on

(iv) Note that $p \wedge q \equiv 1$

$p' \wedge q \equiv 0$

$p \wedge q' \equiv 0$

$p' \wedge q' \equiv 0$

Use

- The activity can be used to explain application of switch connection in series to Mathematical Logic.

ACTIVITY 15

Application of Switch Connection in Parallel with Mathematical Logic

Materials

Switches S_1, S_2, electric wire, battery and lamp.

Preparation

(i) Connect switches S_1, S_2 in parallel.

(ii) Connect battery and Lamp so as to complete the circuit as in the figure 4.3.15.

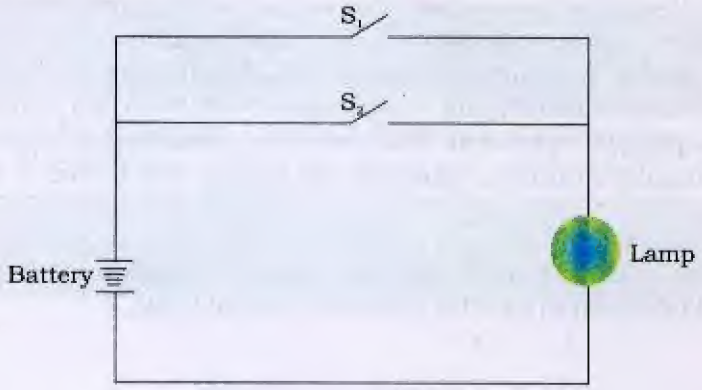


Fig. 4.3.15

Demonstration

Explain that lamp will glow if at least one of the switches is closed as given in the following flow table:

Switch S_1	*Switch S_2*	*Status of Lamp*
Closed	Open	Lamp will glow i.e. current flows
Open	Closed	Lamp will glow i.e. current flows
Open	Open	Lamp will not glow i.e. current does not flow
Closed	Closed	Lamp will glow i.e. current will flow.

(i) Explain that if '+' operation is identified with switches connected in parallel, we obtain the following table.

Switch S_1	*Switch* S_2	*Parallel switch operation*	*Status of Lamp*
1	0	1 + 0	1→ Lamp will glow
0	1	0 + 1	1→ Lamp will glow
0	0	0 + 0	0→ Lamp will not glow
1	1	1 + 1	1→ Lamp will glow

(ii) Explain that $p \vee q \equiv 1$; $p' \vee q \equiv 1$; $p' \vee q \equiv 1$

$p' \vee q' \equiv 0$

(iii) It is interesting to note that logical arrangements made in above stated activities, lead to the formulation of *Boolean* Algebra by taking the set B = {0,1} depicting the states of switches, identifying '+' operation with switches connected in parallel operation, with switches connected in series and complementation operation ' by taking $0' = 1$ and $1' = 0$

Use

- The activity can be used to explain application of switch connection in parallel to Mathematical Logic.

ACTIVITY 16

Graphs of e^x and $\log_e x$

Material

Wooden strips, wire, colour.

Preparation

(i) Fix two perpendicular strips intersecting at a point to represent x – axis and y-axis.

(ii) Draw the graph of e^x by using the values of x and e^x as given below:

x	-2	-1	0	1	2
e^x	.135	.3	1	2.7	7.8

(iii) Draw of graph of $\log_e x$ by using the values of x and $\log_e x$ as given in below:

x	.1	.2	.5	1	e
$\log_e x$	-2.3	-1.6	-.69	0	1

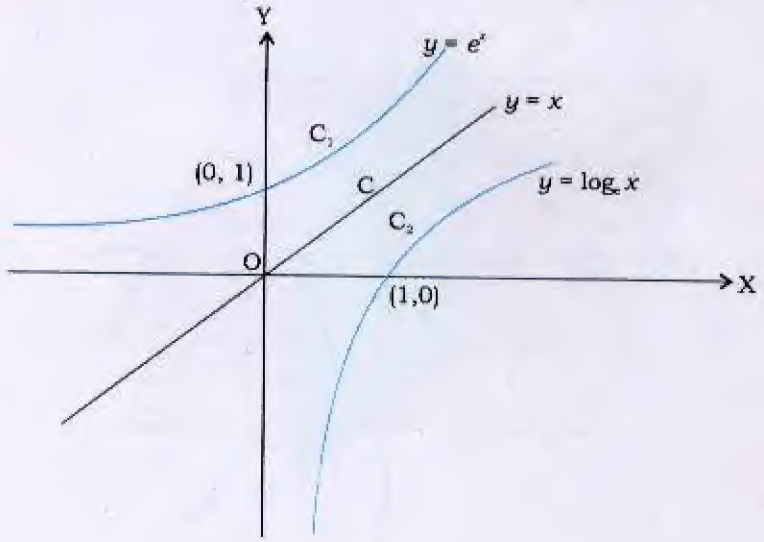


Fig. 4.3.16

(iv) Take wires in the shape of the graphs of e^x and $\log_e x$ and fix them on the wooden strips

(v) Colour the wires with different colours to represent different curve.

Demonstration

(i) Wire C_1 represents e^x

(ii) Wire C_2 represents $\log_e x$.

(iii) Wire C represents $y = x$.

(iv) Wire C_1 crosses y-axis at (0, 1)

(v) Wire C_2 crosses $x-$ axis at (1,0).

(vi) e^x and $\log_e x$ are mirror reflection of each other with respect to $y = x$. They are symmetrical with respect to $y = x$.

Use

- Model can be used to demonstrate the graph of e^x and $\log_e x$.
- Model can be used to compare the shape of e^x and $\log_e x$ with respect to $y = x$.

ACTIVITY 17

Octants

Materials

Thin sheet of wood, wires, wooden board.

Preparation

(i) Take three thin sheets of wood and cut them in size 30 cm × 30 cm.

(ii) Fix two sheets in such a way that they intersect *orthogonally* in the middle of each other (Fig. 4.3.17).

(iii) Cut the third sheet into two equal rectangles.

(iv) Insert one rectangle from one side in the middle cutting the two *orthogonally*, and the other rectangle from other side.

(v) In this model, three planes are intersecting at right angles at a point and they divide the space into eight parts. Each part is called an octant.

(vi) Fix the model on a wooden board.

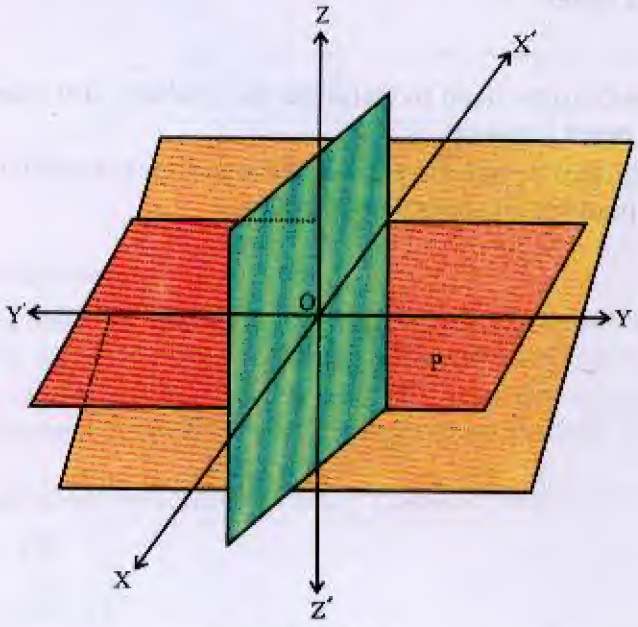


Fig. 4.3.17

Demonstration

(i) In one octant fix scales to show x-axis, y-axis and z-axis. The needle of axis piercing to other side represents XX'. Similarly YY' and ZZ' are represented.

(ii) The point where XX', YY' and ZZ' intersect is origin.

(iii) The distance of point P on XY plane with coordinates (x, y) from the origin is $\sqrt{x^2+y^2}$.

(iv) Fix a rod perpendicular to XY plane at P and parallel to z-axis.

(v) Fix a wire joining the origin to the upper tip P' of perpendicular rod.

(vi) The distance of P' with coordinates (x, y, z) in space from the origin is $\sqrt{x^2+y^2+z^2}$.

(vii) In other octant, if we fix a wire perpendicular to any of the planes, then it will represent normal to the plane.

(viii) If two normals are drawn to any two of the planes, then normals to two perpendicular planes are perpendicular to each other.

Use

- Model can be used to visualise the position and coordinates of a point in space.
- Model can be used to explain the distance between a point in the plane or in space to the origin.

ACTIVITY 18

Vector as a Linear Combination of Vectors

Material

Wooden base and wire.

Preparation

(i) Take three wires of equal length.

(ii) Fix the wire perpendicularly to each other at a point to represent three coordinate axes.

(iii) Taking the three axes as the edges of the cuboid, complete the cuboid with the pieces of wire.

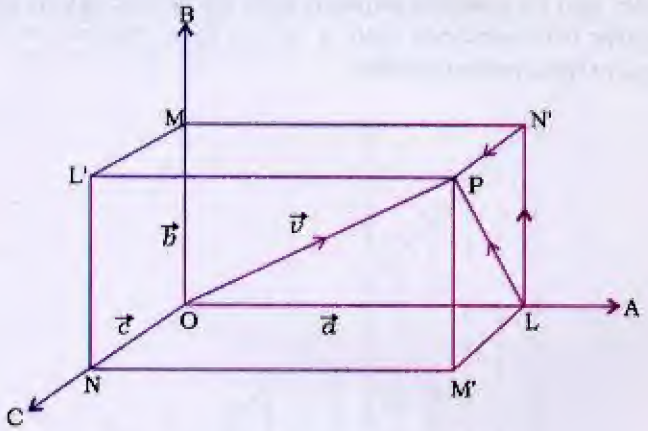


Fig. 4.3.18

Demonstration

(i) The lines OA, OB and OC being non coplanar, taken two at a time determine three different planes BOC, COA and AOB.

(ii) As three vectors OA, OB and OC are non- coplanar, they represent a set of linearly independent vectors.

(iii) Taking $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OC} = \vec{c}$ and $\overrightarrow{OP} = \vec{v}$

(iv) $\vec{v} = \overrightarrow{OP}$

$= \overrightarrow{OL} + \overrightarrow{LP}$

$= \overrightarrow{OL} + \overrightarrow{LN'} + \overrightarrow{N'P}$

$= \overrightarrow{OL} + \overrightarrow{OM} + \overrightarrow{ON}$

(v) $\overrightarrow{OL} = \alpha\overrightarrow{OA} = \alpha\,\vec{a}$ $\overrightarrow{OM} = \beta\overrightarrow{OB} = \beta\,\vec{b}$.

$\overrightarrow{ON} = \gamma\,\overrightarrow{OC} = \gamma\,\vec{c}$

(vi) $\vec{v} = \alpha\,\vec{a} + \beta\,\vec{b} + \gamma\,\vec{c}$

(vii) As $\vec{v}$ is a linear combination of the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$, the four vectors $\vec{v}$, $\vec{a}$, $\vec{b}$, $\vec{c}$, are linearly dependent vectors.

Use

- Model can be used to explain that three vectors in space are linearly independent and a set of four vectors are always linearly dependent vectors.

ACTIVITY 19

Discontinuity of a Function

Materials

Hardboard plate, eighteen 1.5 volt bulbs, one 2.5-volt bulb, 12 testing screws, electrical circuit.

Preparation

(i) Take a hard board plate of size 30 cm × 30 cm.

(ii) Drill nine holes along the horizontal line at equal lengths. Lable them as –5 – 4, –3, –2, –1, 0, 1, 2, 3, 4, 5 and fix bulbs in these holes.

(iii) Drill nine holes along the line, which is inclined at an angle 45° with the horizontal line. Fix bulbs in these holes.

(iv) Complete the electrical circuit.

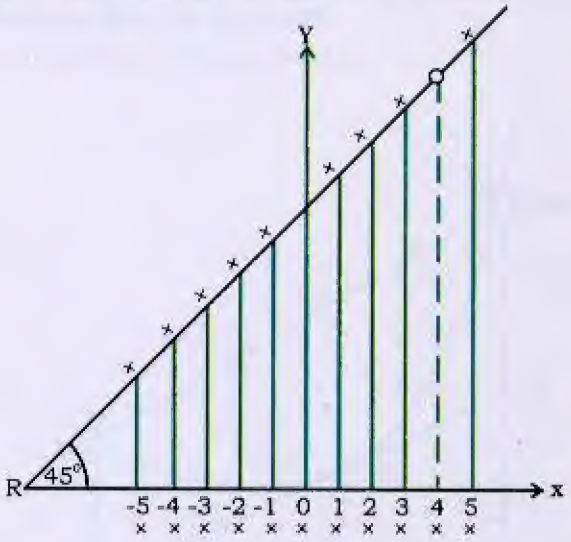


Fig. 4.3.19

Demonstration

(i) Bulbs along the horizontal line represent the points on the x-axis.

(ii) Bulbs along the slope line represent the corresponding points on the graph of function $y = \dfrac{x^2 - 16}{x - 4}$.

(iii) At $x = 4$ the bulb does not glow, i.e., y is not defined and showing that y is discontinuous at $x = 4$.

(iv) In the immediate neighborhood of $x = 4$ (on either side of $x = 4$), the bulb glows and showing that y is continuous in the neighborhood of $x = 4$.

(v) The existence of left hand and right hand limits can be seen by taking different values of x.

Use

- Model can be used to explain the concept of discontinuity of a function at some point.

ACTIVITY 20

Lagrange's Mean Value Theorem

Materials

Plywood, wires, wooden bars.

Preparation

(i) Take two wooden bars of size 16 cm × 20 cm to represent x-axis and y-axis.

(ii) Take a piece of wire of about 25 cm length and bend it in the shape of a curve.

(iii) Fix two wires of 10 cm and 13 cm length at two different points of the curve parallel to y-axis. Join the two end points of the ordinate on the curve by another piece of wire.

(iv) Fix a wire of about 16 cm length at a point of the curve to represent tangent to the curve.

(v) Fix the model or a plywood base.

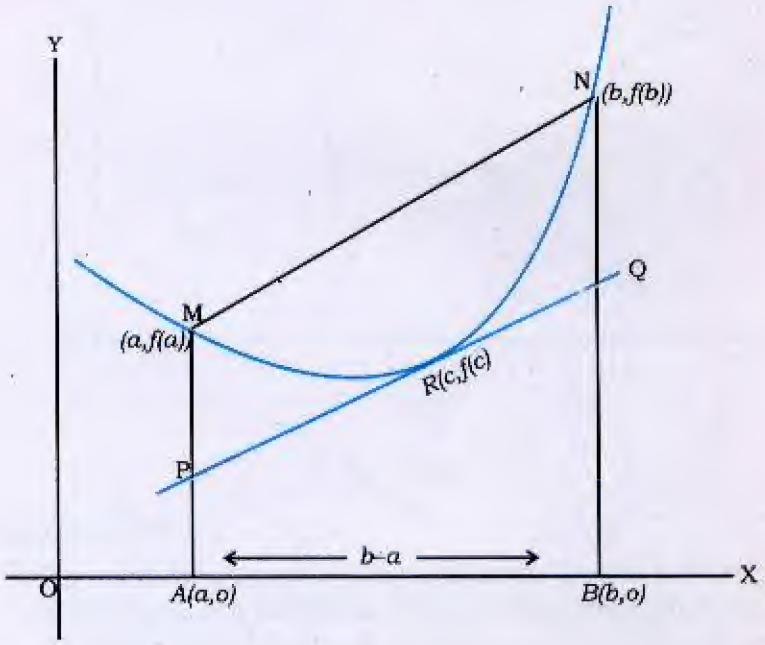


Fig. 4.3.20

Demonstration

(i) MN represents a chord joining end points of the ordinates.

(ii) PQ represents the tangent to the curve at the point $(c, f(c))$, corresponding to the interval (a, b).

(iii) $f'(c)$ represents the slope of the tangent PQ.

(iv) $\frac{f(b)-f(a)}{b-a}$ represents the slope of the chord MN.

(v) MN is parallel to PQ so that

$$f'(c) = \frac{f(b)-f(a)}{b-a}.$$

Use

- Model can be used to explain the *Lagrange's* Mean Value Theorem.

ACTIVITY 21

Maxima and Minima

Materials

Plywood, wires

Preparation

(i) Take two pieces of wires each of 40 cm length to make x-axis and y-axis.

(ii) Take another wire of about 75 cm long and bend it in the shape of a curve to show maximum, minimum points and the point of inflection.

(iii) Take five wires of 2 cm length and fix them at the points of top and bottom bending position of the wire.

(iv) Fix the model on a plywood base.

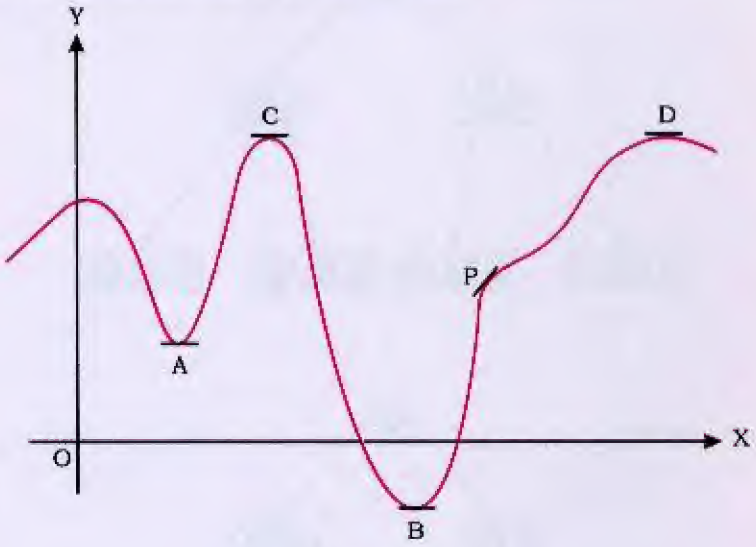


Fig. 4.3.21

Demonstration

(i) The wires representing tangents at the points A, B, C and D to the curve are parallel to x-axis which means that the first derivative of the function representing the curve is zero at these points.

(ii) At the points A and B, sign of the first derivative changes from –ve to +ve, so these are the points of local minima.

(iii) At the points C and D, sign of the first derivative changes from +ve to –ve, so these are the points of local maxima.

(iv) At the point P, shape of the curve changes but sign of the derivative does not change, so P is a point of inflection.

Use

- Model can be used to explain the concepts of points of local maxima, local minima and inflection.

ACTIVITY 22

Probability

Material

Cardboard, chart paper, metal discs, marker, adhesive.

Preparation

(i) Take a piece of cardboard and cover it with the chart paper.

(ii) Fix metal discs on the chart paper, for showing different sample spaces and events.

(iii) Take a marker and write 'H' and 'T' on the discs, where H stands for 'head' and T stands for 'Tail' on the metal discs.

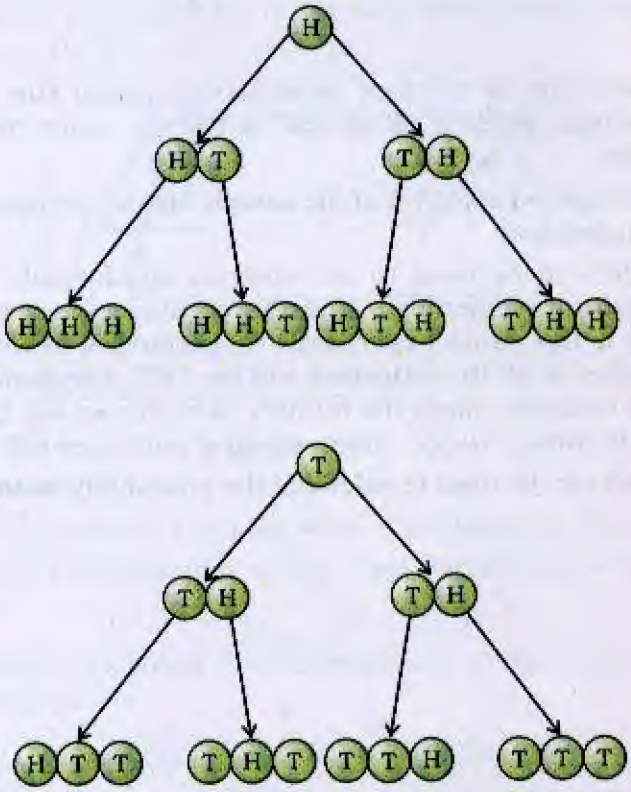


Fig. 4.3.22

Demonstration

Let S denote the sample space. Then S can be demonstrated in the following manners.

(i) If a disc is thrown once, the sample space S = {H, T}

(ii) If a disc is thrown twice, the sample space S = {HT, TH, HH, TT}

(iii) If a disc is thrown thrice, the sample space S = {HHH, HTT, THT, TTH, HHT, HTH, THH, TTT}

(iv) Take A as a subset of S such that it has at least one head, then

A = {HTT, THT, TTH, HHT, HTH, THH, HHH}

If A denotes an event, then $P(A) = \frac{7}{8}$.

Use

(i) Model can be used to demonstrate about the possible outcomes when a metal disc is thrown once, twice and thrice.

(ii) Model gives exact idea of the sample space, events and their probabilities.

(iii) Model can be used to demonstrate the formula – "If we perform an experiment and the number of outcomes is x and if the same experiment is performed n times, the number of all the outcomes will be "x^n". For example, if a coin is thrown once, the number of outcomes will be 2, But if it is thrown thrice, the number of outcomes will be 2^3.

(iv) Model can be used to calculate the probability of an event.

ACTIVITY 23

Geometrical Interpretation of Scalar Triple Product [i.e. $\vec{a}.(\vec{b}\times\vec{c})$]

Materials

Wires, soldering wire, wire cutter.

Preparation

(i) Cut four wires of about 20 cm length to show length of $\vec{a}$.

(ii) Cut four wires of about 25 cm length to show length of $\vec{b}$.

(iii) Cut four wires of about 15 cm length to show length of $\vec{c}$.

(iv) Construct a parallelopiped with the help of a, $\vec{b}$ and $\vec{c}$.

(v) Fix a wire perpendicular to the base parallelogram to show length of $\vec{b}\times\vec{c}$

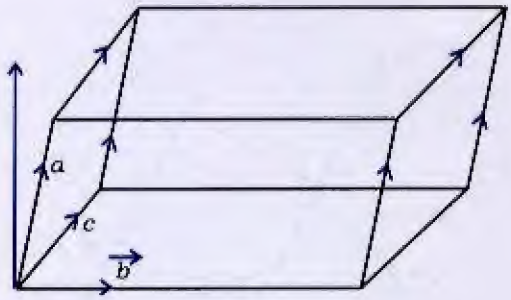


Fig. 4.3.23

Demonstration

(i) $\vec{b}$ and $\vec{c}$ represent adjacent sides of the base parallelogram.

(ii) The line perpendicular to the base parallelogram is along $\vec{b}\times\vec{c}$

(iii) Projection of $\vec{a}$ along $\vec{b}\times\vec{c}$ corresponds to the height of the prallelopiped.

(iv) $\vec{a}.(\vec{b}\times\vec{c})$ represents volume of the parallelopiped.

Use

- Model can be used to interpret scalar triple product of vectors geometrically.

ACTIVITY 24

Equation of a Straight Line in Space passing through a Fixed Point and Parallel to a given Vector $\vec{m}$.

Material

Wooden plank, wires.

Preparation

(i) Take a wooden base of 7cm × 6 cm.

(ii) Take two wires C_1 and C_2 to represent arbitrary vector $\vec{r}$ and vector $\vec{a}$ through the origin in the space.

(iii) Fix a wire C_3 as shown in the figure to represent the given line.

(iv) Fix a wire C_4 to represent the direction of the vector $\vec{m}$.

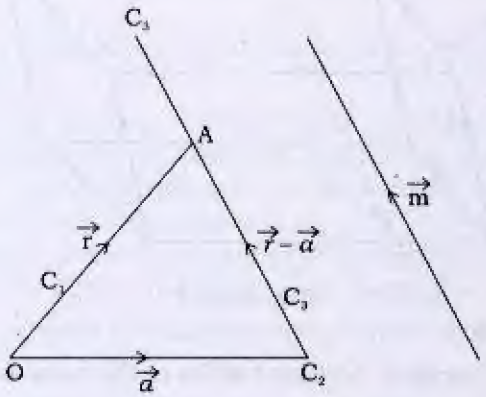


Fig. 4.3.24

Demonstration

(i) C_1 and C_2 wires represent vectors $\vec{r}$ and $\vec{a}$

(ii) C_3 wire represents the straight line parallel to the given vector $\vec{m}$

(iii) C_4 wire represents the given direction of the vector .

(iv) C_3 represents the vector $\vec{r}-\vec{a}$ parallel to vector $\vec{m}$ so that $\vec{r}-\vec{a}=\lambda\vec{m}$ i.e. $\vec{r}=\vec{a}+\lambda\vec{m}$

Use

- Model can be used to explain for a line in space passing through two fixed points with position vectors $\vec{a}$ and $\vec{b}$ and parallel to a given vector $\vec{m}$

ACTIVITY 25

Equation to the Plane Passing Through a Fixed Point and Perpendicular to the Normal, i.e., $(\vec{r} - \vec{a}).\ \vec{n} = 0$

Material

Wooden plank of size of about 8cm ×10cm, transparent sheet, wires, soldering material.

Preparation

(i) Take a piece of wood of size 8cm ×10cm.

(ii) Take a piece of thin wooden rod and fix it in the wooden plank to represent the normal $\vec{n}$.

(iii) Take three pieces of wires to represent vectors $\vec{r}$, $\vec{a}$ and $\vec{r} - \vec{a}$.

(iv) Join three wires in the shape of a triangle.

(v) Join the triangle with transparent sheet in such a way that one side touches the sheet and the other two sides of the triangle are in space and vertex at the bottom of wooden plank as shown in the figure 4.3.25.

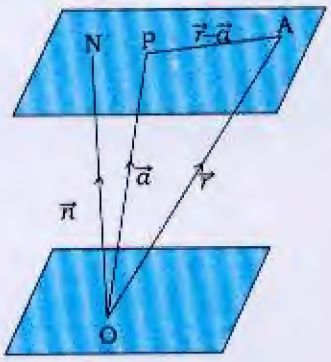


Fig. 4.3.25

Demonstration

(i) O represents the origin on the base.

(ii) $\vec{r}$ represents position vector of A with reference to the origin.

(iii) $\vec{a}$ represents position vector of point P with reference to the origin.

(iv) $\vec{PA}$ represents vector $\vec{r} - \vec{a}$.

(v) Vertical wooden rod represents normal to the plane.

(vi) $(\vec{r} - \vec{a}).\vec{n} = 0$ represents vector equation of plane as the two vectors $\vec{r} - \vec{a}$ and $\vec{n}$ are perpendicular and their dot product is zero.

Use

Model can be used to demonstrate

- The position vectors of different points.
- Vector.
- Vector normal to the plane.
- Equation of plane.

ACTIVITY 26

Shortest Distance between Two Lines

Materials

Wooden base, wooden blocks, wires, adhesive.

Preparation

(i) Take a wooden base of size 30 cm × 8 cm.

(ii) Take four wooden blocks of size 10 cm × 8 cm × 3 cm, 18 cm × 8 cm × 3 cm, 8 cm × 8 cm × 3 cm and 15 cm × 8 cm × 3 cm.

(iii) Fix two blocks of size 10 cm × 8 cm × 3 cm and 10 cm × 8 cm × 3 cm on one side of the base.

(iv) Fix other two remaining blocks on the opposite side of the base.

(v) Join lower block in the left hand side to the upper block in the right hand side by a wire AB.

(vi) Join upper block in the left hand side to the lower block in the right hand side by a wire CD.

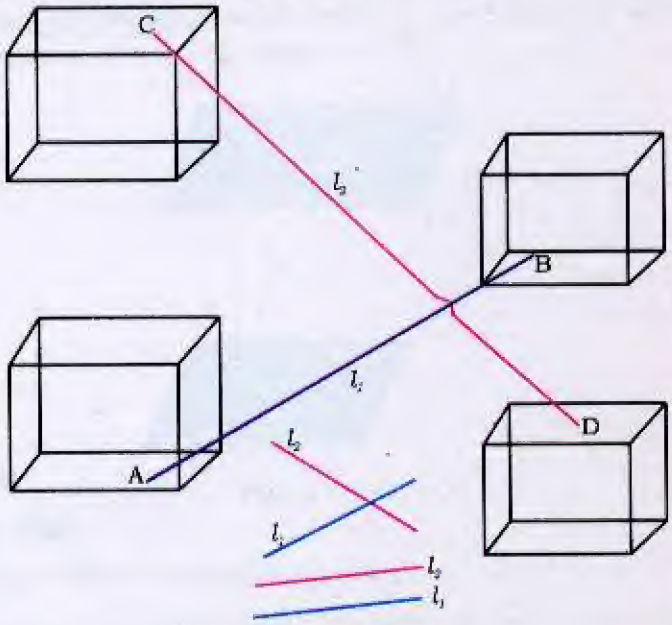


Fig. 4.3.26

Demonstration

(i) AB and CD represent lines l_1 and l_2 which are *skew* lines.

(ii) d is the shortest distance between the skew line l_1 and l_2.

(iii) d is the shortest distance between two lines l_1 and l_2 and is perpendicular to both l_1 and l_2.

(iv) If $d = 0$, the lines are intersecting.

(v) If l_1 and l_2 are in the same plane, then they are either parallel or intersecting.

Use

- Model can be used to explain the concept of shortest distance between two lines in space.

ACTIVITY 27

Angle between Two Planes

Material

Plywood pieces, wires, hinges.

Preparation

(i) Take two pieces of plywood of size 5cm × 5cm and join them with the help of hinges.

(ii) Fix two vertical wires on each plane to show normals to the planes.

(iii) Cut slots in the two planes to fix a third plane.

(iv) Take the third plane and fix it in the slots of two planes as shown in the figure.

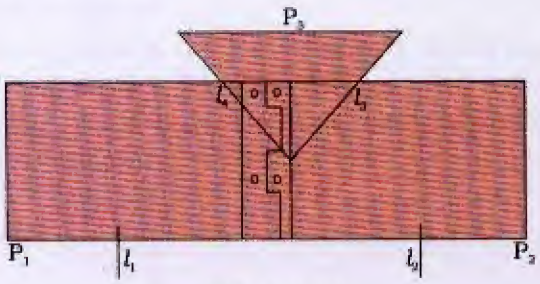


Fig. 4.3.27

Demonstration

(i) P_1 represents the first plane.

(ii) P_2 represents the second plane.

(iii) Two vertical wires l_1 and l_2 represent normals to the planes P_1, P_2, respectively.

(iv) l_3 and l_4 are the lines of intersections of the plane P_3 with P_1 and P_2, respectively.

(v) Angle between lines l_3 and l_4 is the angle between the planes. It is same as the angle between their normals as shown in the figure.

Use

- Model can be used to explain that the angle between two planes is the same as the angle between their normals.

ACTIVITY 28

Distance of a Point form a Plane

Materials

Wooden board, hinges, iron rod.

Preparation

(i) Take a wooden board of size 22 cm × 18 cm.

(ii) Take another piece of wooden board of size18 cm × 15 cm.

(iii) Connect the two boards by hinges.

(iv) Take a triangular lamina and fix it in the board in a groove.

(v) Take a point on the wooden board and fix a vertical rod of height equal to the distance between two parallel planes.

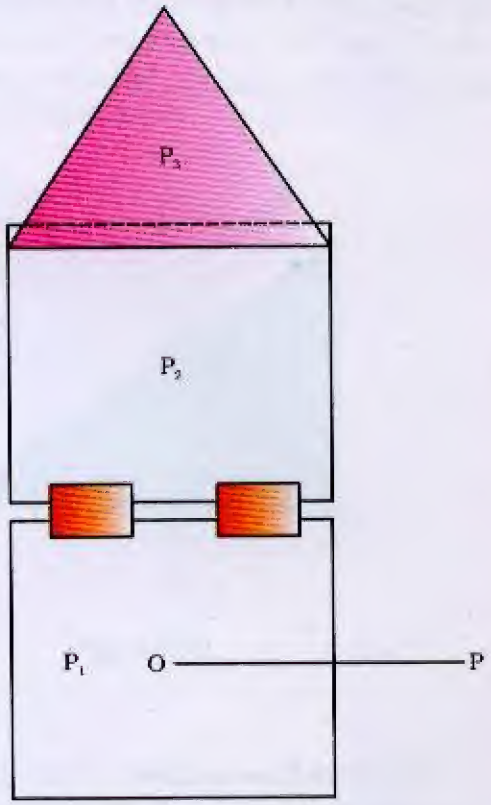


Fig. 4.3.28

Demonstration

(i) P_1 represents the horizontal plane.

(ii) P_2 represents the vertical plane.

(iii) P_3 represents triangular lamina parallel to the plane P_1.

(iv) Vertical rod OP represents normal to the plane P_1 with tip P of the rod as a point.

(v) To show the distance of point P from plane P_1, the triangular lamina P_3 is brought in such a way that it passes through the tip P. The length of the rod from O to P represents the distance between a point from a plane.

Use

- Model can be used to explain the concept of distance of a point from a plane.

ACTIVITY 29

Application of the Lami's Theorem

Material

Plywood, hinges, wire, wooden weight, thread.

Preparation

(i) Take a thin plywood piece of size 1cm × 1cm for making the base of the model.

(ii) Take a wooden plane in the shape of a right angled triangle.

(iii) Make a stand for this plane by wooden support.

(iv) Hang a wooden weight W on the plane with the help of thread.

(v) Support the weight W by other force T represented by wire which is inclined at an angle β with the vertical line.

(vi) Take wire through P making an angle α with the vertical line to show resultant R of W and T.

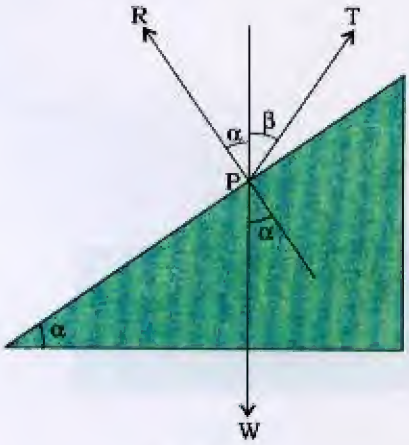


Fig. 4.3.29

Demonstration

(i) W is force represented by hanging wooden weight.

(ii) T represents support of W, inclined at an angle β with the vertical.

(iii) R is the resultant force inclined with the vertical line at the same angle as the inclination of the plane with the horizontal line

(iv) α is the inclination of the plane with the horizontal line.

Use

- Model can be used to illustrate Lami's Theorem.

ACTIVITY 30

Centre of Parallel Forces

Material

Thick wooden rod, two vertical and three horizonal wooden rods, wires, hinges, small wooden weights.

Preparation

(i) Take a wooden rod of size 1.5 cm × 8 cm for the base of the model.

(ii) Take two vertical wooden support of size 1cm × 2 cm.

(iii) Take one horizontal wooden rod of size 1.5 cm × 2 cm to support the line of action of forces made by wire.

(iv) Take two wooden rods of equal length and hang them at G_1 and G_2 as shown in the figure 4.3.30.

(v) Hang the wooden weights P_1, P_2, P_3 and P_4 at the hinges with the help of thread.

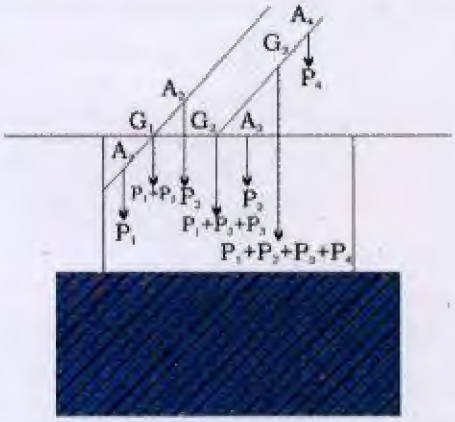


Fig. 4.3.30

Demonstration

(i) P_1 is the force acting at A_1.

(ii) P_2 is the force acting at A_2.

(iii) $P_1 + P_2$ is resultant of forces P_1 and P_2 acting at G_1.

(iv) $P_1 + P_2 + P_3$ will be the resultant of forces at G_1 and A_3 and will be acting at G_2.

(v) $P_1 + P_2 + P_3 + P_4$ will be resultant of forces at G_2 and A_4 and will be acting. at G_3. G_3 will be the centre of like parallel forces P_1, P_2, P_3 and P_4.

Use

- Model can be used to explain that the centre of parallel forces is a certain unique point through which the resultant of a given system of parallel forces passes.

ACTIVITY 31

Moment of a Force about a Point

Material

Thin plywood pieces, hinges, wire.

Preparation

(i) Take a thin plywood piece of size 8″ × 8″.

(ii) Take another plywood piece of size 8″ × 3″

(iii) Connect the plywood pieces by hinges.

(iv) Take a point P on the bigger plywood piece.

(v) Connect the point P to point O on the axis of rotation with the help of a wire to represent vector $\vec{r}$.

(vi) Fix a wire through the point P which is not in the bigger plane to represent the force $\vec{F}$.

(vii) Fix another wire perpendicular to the plane to represent the moment of force as $\vec{r} \times \vec{F}$.

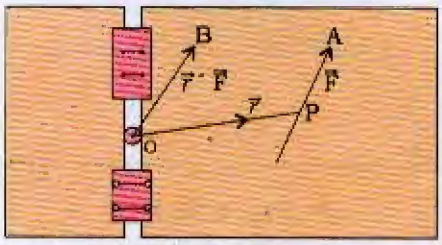


Fig. 4.3.31

Demonstration

(i) The vector $\overrightarrow{OP}$ from the point O to P represents the vector $\vec{r}$

(ii) The vector $\overrightarrow{PA}$ from the point P to A represent the vector $\vec{F}$.

(iii) The vector $\overrightarrow{OB}$ from O to B represent the vector $\vec{r} \times \vec{F}$ which gives the moment of force $\vec{F}$ about point O. The nature of this moment of force will be to rotate the bigger plane about the axis.

Use

- Model can be used to explain moment of a force about a point.

ACTIVITY 32

Projectile Motion

Materials

Wooden frame, wire, hinges.

Preparation

(i) Take two perpendicular intersecting wires to represent x-axis and y-axis.

(ii) Make a trajectory with the help of wire.

(iii) Fix a wire at initial point of the trajectory to show the tangent.

(iv) Fix another tangential wire at any point of the trajectory to show direction of velocity.

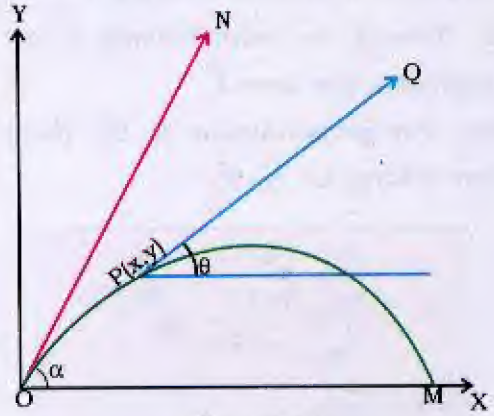


Fig. 4.3.32

Demonstration

(i) OX and OY are coordinate axes.

(ii) The tangent ON represents the direction of initial velocity.

(iii) The tangent PQ represents the direction of velocity of the particle after time t at the position P(x, y).

(iv) Path OM represents the path of the projectile.

(v) α is the angle of projection.

Use

- Model can be used to visualize a projectile motion.

ACTIVITY 33

Normal Probability Curve and Area under the Normal Probability Curve

Material

Wires, plywood, adhesive.

Preparation

(i) Take two pieces of wires of length 30 cm to make x-axis and y-axis.

(ii) Take a piece of wire of length 50 cm and bend it in the shape of a curve to show normal probability curve.

(iii) Take two pieces of wires of same size and fix them with the normal probability curve.

(iv) Fix the model on the plywood base.

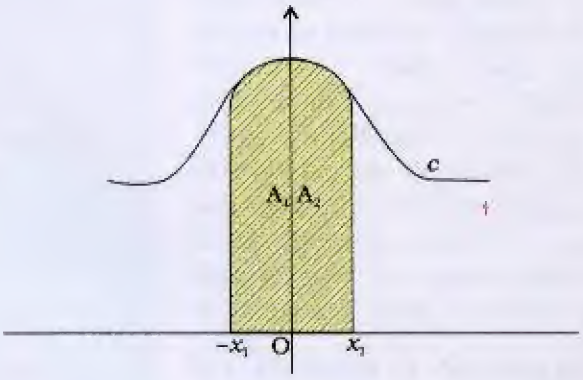


Fig. 4.3.33

Demonstration

(i) c represents normal probability curve

(ii) The region A_1 and A_2 represent area of the region between $x = x_1$ and $x = -x_1$.

(iii) The area of the region between $x = -x_1$ and $x = 0$ is equal to the area of the region between $x = 0$ and $x = x_1$.

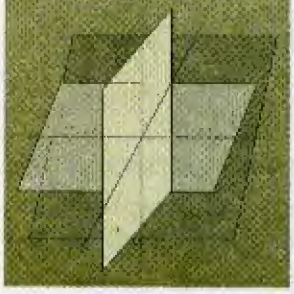
(iv) The height of the curve decreases as we proceed to either direction from the mean.

Use

Model can be used to demonstrate:

- the normal probability curve
- the area under normal probability curve.
- that the range of the normal probability curve is unlimited in either direction; but it never touches x-axis.
- that area under the normal probability curve can be found out by using normal distribution table.

5 Projects



Project work in mathematics may be performed individually by a student or collectively by a group of students. These projects may be in the form of construction such as curve sketching or drawing of graphs, etc. It may offer a discussion of a topic from history of mathematics involving the historical development of the topics or concepts. Students may be allowed to select the topics of their own choice for mathematics projects. The teacher may act as a facilitator by creating interest in various topics. Once the topic has been selected, the student should read as much about the topic as is available and finally prepare the project. Some suggested projects are given below:

5.1 Projects on History of Mathematicians

It may include history of Indian mathematicians such as Aryabhata, Brahmagupta, Varahmihir, Sridhara, Bhaskaracharya, Ramanujan etc., and history of foreign mathematicians such as Cantor, Pythagoras, Thales, Euclid, Appollonius, Descartes, Fermat, Leibnitz, Alkhawarizmi, Euler, Fibonacci, Gauss, Newton, etc.

5.2 MATHEMATICS PROJECTS ON SOME MORE TOPICS

5.2.1 Ancient number systems and algorithms.

5.2.2 Finding the number names and numeral of numbers, say 1 to 20 in any five or more different languages.

5.2.3 Collecting objects of congruent shapes from the environment.

5.2.4 Number game by considering different questions for different numbers.

5.2.5 Geometric Euclid constructions.

5.2.6 Pythagorean triplets.

5.2.7 On linear programming problem related to day-to-day life, like collecting data from families of their expenditures and requirements from the factories to maximum output.

5.2.8 Collect data from dieticians, transporters, agents and formulate linear programming problem.

5.2.9 Collection of data from a firm about the number of partners, the ratio of their invested capitals and shared profit.

5.2.10 Chart showing different formulae of annuity.

5.2.11 Make a chart of the formulae of applications of calculus in commerce.

5.2.12 Probability distribution of number of heads in four/five throws of a coin.

5.2.13 Collect data of rates of five years of different day-to-day commodities and calculate the price index.

5.2.14 Mathematics and physics: application of conic sections, vectors three dimensional geometry, calculus etc.

5.2.15 Mathematics and chemistry: study structure of organic compounds.

5.2.16 Mathematics and Biology: study of science of heredity etc.

5.2.17 Mathematics and Music

5.2.18 Mathematics and Environment

5.2.19 Mathematics and Arts: construction of shapes using curve

5.2.20 Mathematics and Information and communication technology: writing of mathematical programmes, flow charts, algorithm, circuit diagrams etc.

5.2.21 Collection of statistical data and analysing it for standard deviation and mean deviation.

5.3 PROJECTS ON RECREATIONAL MATHEMATICS

Students may prepare small project work on the following:

5.3.1 Forming 3×3, 4×4 magic squares with numbers from 1 to 9 and 1 to 16 respectively.

5.3.2 Aryabhata's method of constructing a 3×3 magic square.

5.3.3 Aryabhata's method of constructing a 5×5 magic square.

5.3.4 Geometric dissections such as Tangrams.

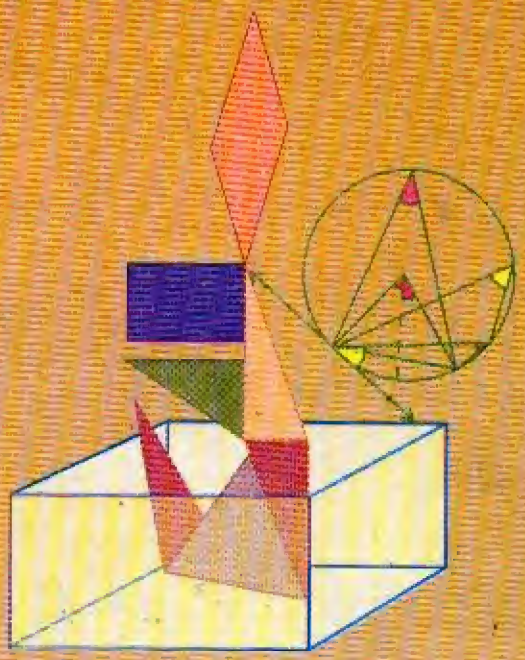
5.3.5 Tesselations

5.3.6 Mathematical Embroidery (Fun with Lines)

5.3.7 Number Patterns

6 Evaluation

Evaluation is an essential component in our teaching-learning process. Present day teaching is based only on chalk and talk method which is without any integration of activity/demonstration work. The greater the integration of activities, the better the outcomes of meaningful learning. Hence activity based teaching has to be so designed so that it can be used as a powerful means of influencing the quality of what teachers teach and what the students learn. This could be possible if some marks for practicals in mathematics are assigned. While emphasising the need of exploration of mathematical facts through experimentation, the *National Curriculum Framework for School Education — 2000* states, *In terms of scheme of evaluation of such mathematics learning, this has to be given weightage equal to that in science.* Therefore, in the opinion of the authors and other subject specialists, about 20% marks of the total marks should be kept for the practical work at Elementary and Secondary stages, and 30% marks at Higher Secondary Stage. These marks may be properly distributed towards projects, viva-voce and doing activity at different stages of school education.



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